Analysis of $B^0_d \rightarrow K^{*0} \mu^+ \mu^-$ Decay with the ATLAS Experiment

PhD Thesis

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Abstract

**ATLAS** is a general-purpose experiment at the Large Hadron Collider. Beside other goals, it also aims at the study of B-hadrons. B-physics offers a large number of channels that can provide information about some fundamental properties of our universe. Among them, the $B^0_d \rightarrow K^{*0}\mu^+\mu^-$ decay is sensitive to the potential presence of particles that are not predicted by the Standard Model. Such “new physics” effects can be observed indirectly by studying angular distributions of the $B^0_d \rightarrow K^{*0}\mu^+\mu^-$ decay products.

This thesis describes the analysis of 4.9 fb$^{-1}$ of data produced in the proton-proton collisions at the centre-of-mass energy $\sqrt{s} = 7$ TeV at the **LHC** in the year 2011 and recorded by the **ATLAS** detector. The main steps of analysis are described, such as the selection of the signal events, the data fit procedure and the estimation of uncertainties. The obtained results are compared with other experiments and with the Standard Model prediction.
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Chapter 1

Introduction

Particle physics is the area of science that studies the fundamental components of matter and their interactions. The theory that has been the most successful in describing this topic is the Standard Model (SM) of particle physics. It incorporates electromagnetic, weak and strong interactions between the elementary particles. It has explained and predicted a number of phenomena, among them are the existence of the top quark, τ-neutrino, and the recently observed Higgs boson [1], [2]. So far no experimental observation contradicts the predictions of the SM. Despite its success, the SM does not offer any explanation for a number of observations, like the neutrino oscillations and their masses, matter-antimatter asymmetry in the universe and the existence of Dark Matter. It also does not include a theory of gravitation. It is therefore clear that the SM alone is not sufficient to describe all the properties of our world, and a more complete theory is required. There are various other theories (Super Symmetry (SUSY) [3] being perhaps the most known and well-developed of them) that attempt to explain physics Beyond the Standard Model (BSM), that are neither excluded nor confirmed by observations. In order to get a hint whether any of them is correct, it is important to look at the processes where deviations to the SM predictions could occur. The deviations, if any, are most likely to be small, and might be found in the rare processes. The search for BSM effects may be conducted in two ways: by trying to directly create new particles in high energy collisions or by looking at the indirect effects of their possible presence in the intermediate stages of various decays.

The field of B-physics offers a number of channels for an indirect search of BSM effects. The decays of interest can only proceed via loop level diagrams and therefore have a small branching ratio. For some of them, like the $B_s \rightarrow \mu^+\mu^-$ decay that has been recently observed by the LHCb [4] and CMS [5] experiments, the value of the branching ratio itself serves as a test of the SM. For others, for instance $b \rightarrow s\mu^+\mu^-$ decays, the relevant information can be obtained from the angular distributions of the
The angular analysis of $B^0_d \rightarrow K^*\mu^+\mu^-$ decay based on 4.9 $fb^{-1}$ of data collected by the ATLAS experiment at the Large Hadron Collider (LHC) in the year 2011 [6] is in the main focus of this thesis. This channel has also been studied by several other experiments, such as Belle [7], BaBar [8, 9], CDF [10], CMS [11] and LHCb [12].

In Chapter 2 of this thesis the theoretical background is outlined. Chapter 3 gives an overview of the LHC and the ATLAS experiment. Chapter 4 is dedicated to the used software and Monte-Carlo (MC) simulations that play a very important role in nearly every physics analysis. Chapter 5 describes the event selection and the details of the data analysis. In Chapter 6 the results are discussed.
Chapter 2

Theoretical Background

In this chapter, an introduction to the main ideas and the mathematical description of the SM is given. In Section 2.2 and 2.3 the decay suppression mechanism is explained. The framework of Effective Field Theory (EFT) is presented, with the focus on $b \to s l^+ l^-$ transitions. The proton-proton collision process from the theoretical point of view is considered in Section 2.5. The motivation for B-physics research and $B^0_d \to K^{*0} \mu^+ \mu^-$ decay in particular are mentioned. The $B^0_d \to K^{*0} \mu^+ \mu^-$ decay geometry is described, the angular observables are discussed and the effects of BSM physics are mentioned.

2.1 Standard Model

The Standard Model (SM) of particle physics is a local gauge Quantum Field Theory (QFT) that describes electromagnetic, weak and strong interactions between the elementary components of matter. A detailed introduction to the SM can be found in [13–15] and other sources.

2.1.1 SM Constituents and Interactions

The elementary particles in the SM can have a spin of 0, 1 or 1/2. The only scalar (spin-0) particle in the SM is the Higgs boson. Particles with spin-1/2 obey the Fermi-Dirac statistics and are called fermions, while those with integer spin obey the Bose-Einstein statistics and are called bosons. The spin-1 particles: photon, $Z^0$ and $W^\pm$ bosons and gluons - are called gauge bosons. Photons are massless and mediate the electromagnetic interaction between the charged particles, $Z^0$ and $W^\pm$ have masses of roughly 91 MeV and 80 MeV, respectively [16], and mediate the weak interaction. Eight massless gluons mediate the strong interaction between the color charged particles (i.e. themselves and the quarks).

The two categories of fermions are quarks and leptons, each including three generations. Inside the generations (or families) the fermions are combined into doublets, as shown in Table 2.1. There are
CHAPTER 2. THEORETICAL BACKGROUND

six quarks: up, down, charm, strange, top, bottom - and their antiparticles. Quarks are massive, they carry electric and color charge, and participate in all types of interactions. Each quark doublet has an up-type quark with electric charge \(+2/3\) (in units of the electron charge) and a down-type quark with the charge \(-1/3\). Due to the color confinement phenomenon quarks are bound to each other, forming colorless states - hadrons: mesons containing a quark and an antiquark and (anti-)baryons consisting of three (anti-)quarks.

Among leptons, the neutrinos are neutral and can interact only by the weak force, while electron, muon and \(\tau\)-lepton also have an electric charge and therefore interact both weakly and electromagnetically. Leptons are grouped into doublets according to their leptonic charge - \(L_e\), \(L_\mu\) or \(L_\tau\).

The fundamental particles and their interactions are schematically presented in Figure 2.1.

<table>
<thead>
<tr>
<th>Type</th>
<th>Generations</th>
<th>Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I   II</td>
<td>III</td>
</tr>
<tr>
<td>Quarks</td>
<td>u   c</td>
<td>t</td>
</tr>
<tr>
<td></td>
<td>d   s</td>
<td>b</td>
</tr>
<tr>
<td>Leptons</td>
<td>e(^-)</td>
<td>(\mu^-)</td>
</tr>
<tr>
<td></td>
<td>(\nu_e)</td>
<td>(\nu_\mu)</td>
</tr>
</tbody>
</table>

Table 2.1: Fermions of the Standard Model.

Figure 2.1: Fundamental particles and interactions [17].
2.1. Standard Model

2.1.2 Group Formalism

The three types of fundamental interactions can be described by the corresponding symmetry groups [18]. The group $SU(3)_C$, where index $C$ stands for color charge, generates the strong interactions, local invariance under $SU(3)_C$ corresponds to the conservation of the color charge. Electroweak interactions are generated by the tensor product $SU(2)_L \otimes U(1)_Y$. Local invariance under $U(1)_Y$ group leads to the conservation of the weak hypercharge $Y = 2(Q - I_3)$, where $I_3$ is the third component of the weak isospin and $Q$ is the electrical charge. Local invariance under $SU(2)_L$, where index $L$ refers to left-handed fermions, means the conservation of the weak isospin. As a gauge theory, the SM must be invariant under the transformations of the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ group. Generators of the group $SU(3)$ are Gell-Mann matrices $\lambda_i$ with the commutation relation

$$[\lambda_i, \lambda_j] = 2i f_{ijk} \lambda_k, \quad (2.1)$$

where $f_{ijk}$ are the structure constants of the group.

Generators $T_i$ of the group $SU(2)$ are proportional to the Pauli matrices $\sigma_i$ ($T_i = \sigma_i / 2$) and obey the following commutation relation:

$$[T^i, T^j] = 2\epsilon_{ijk} T^k, \quad (2.2)$$

where $\epsilon_{ijk}$ is the antisymmetric Levi-Civita tensor.

Group $U(1)$ describes all complex numbers with unitary absolute value.

2.1.3 Lagrangian of the Standard Model

Let us now outline the components necessary to construct the Lagrangian $L$ of the SM. In the QFT, one usually considers the Lagrangian density $\mathcal{L}$, such that

$$L = \int \mathcal{L} d^3x. \quad (2.3)$$

Due to the gauge invariance and renormalizability, the mass terms cannot be explicitly included in the Lagrangian. However, the gauge bosons must be massive, since the weak interaction is short-ranged, so the gauge invariance must be broken spontaneously [19].

The SM Lagrangian can be represented as a sum of four terms [20]:

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} \quad (2.4)$$

Here, the kinetic terms for all fermions and gauge bosons as well as terms responsible for the self-interactions of the gauge bosons are included in $\mathcal{L}_{\text{gauge}}$. The term $\mathcal{L}_{\text{int}}$ describes the interactions between
gauge bosons and fermions. The dynamics of the Higgs field is contained in $\mathcal{L}_{\text{Higgs}}$, while $\mathcal{L}_{\text{Yukawa}}$ describes the Yukawa couplings of fermions with the Higgs field.

Three types of interactions between electroweak gauge bosons and SM fermions can be distinguished: charged current, neutral current and electromagnetic current. The weak charged current describes the coupling of fermions to $W$ bosons, the neutral current - to $Z^0$ boson and the electromagnetic current - to photon. In the tree level of neutral-current reactions, the change of fermions flavor is forbidden - in other words, the neutral current mediated through gauge bosons $Z^0$, $\gamma$, $g$ does not change flavor. The Flavor Changing Neutral Currents (FCNC) may occur only at the loop level, which makes such processes very rare. The study of FCNC reactions can provide a good test of the SM and give strong limitations to the BSM theories.

### 2.2 GIM Mechanism

According to the Cabibbo principle [21], the weak interaction does not couple to the mass eigenstates of the quarks, but to their linear combination. In the 1960s, only three quarks were known, and Cabibbo represented the quark mixing as a combination

$$d_C = d \cos \theta + s \sin \theta. \quad (2.5)$$

The weak current could then be written as

$$J_\mu = u\gamma_\mu(1 - \gamma_5)d_C. \quad (2.6)$$

However, this would allow for FCNC decays at the tree level, which was excluded experimentally by the non-observation of $K_L \rightarrow \mu^+\mu^-$ decay. To resolve this contradiction, in 1970 Sheldon L. Glashow, John Iliopoulos and Luciano Maiani proposed to add a fourth quark, that would be coupled by the weak interaction to the linear combination of $d$- and $s$-quarks orthogonal to the one constructed by Cabibbo [22]:

$$s_C = -d \sin \theta + s \cos \theta \quad (2.7)$$

The full charged weak current then became

$$J_\mu = u\gamma_\mu(1 - \gamma_5)d_C + c\gamma_\mu(1 - \gamma_5)s_C. \quad (2.8)$$

In matrix form,

$$J_\mu = U\gamma_\mu(1 - \gamma_5)CD \quad (2.9)$$
2.3. **CKM Matrix**

The Cabibbo-Kobayashi-Maskawa (CKM) matrix is a unitary $3 \times 3$ complex matrix representing the extension of the Cabibbo formalism to the three quark generations [23]. Its elements provide the information on the strength of flavor-changing weak processes.

Equation 2.9 for the weak current is still valid in the case of three quark families, but instead of 2×2 Cabibbo matrix $C$ the 3×3 CKM matrix $V$ must be used:

$$ V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. $$

The transition probability between quark $i$ and quark $j$ is defined by value of the corresponding matrix element $V_{ij}$. Latest measurements give the following values of the CKM matrix elements [16]:

$$ V = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}. $$

(2.12)
From these numbers, the effect of the suppression can be clearly seen. Since CKM matrix is unitary, the following equations are valid for all generations $i$:

$$
\sum_k |V_{ik}|^2 = \sum_i |V_{ik}|^2 = 1 \quad (2.13)
$$

and

$$
\sum_k V_{ik}V_{jk}^* = 0. \quad (2.14)
$$

The unitary $3 \times 3$ matrix can be described with 9 parameters: three angles and six complex phases. Five of these phases do not correspond to any physical observables. Therefore, four independent parameters are left that can be chosen arbitrarily.

### 2.3.1 CKM Matrix Parametrizations

There are several ways to choose the four parameters in the CKM matrix. Three most common of them include the original parametrization used by Kobayashi and Maskawa [23], the “standard” parametrization [24] and Wolfenstein parametrization [25]. The first one uses three angles $\theta_1, \theta_2, \theta_3$ and a Charge Parity Symmetry (CP)-violating phase $\delta$. The matrix becomes

$$
V = \begin{pmatrix}
    c_1 & s_1c_3 & -s_1s_3 \\
    s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\
    s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta}
\end{pmatrix}. \quad (2.15)
$$

Here $c_i = \cos \theta_i$ and $s_i = \sin \theta_i$. In the standard parametrization, the four parameters are Euler angles $\theta_{12}, \theta_{13}, \theta_{23}$ and one CP-violating phase $\delta_{13}$, and the matrix is

$$
V = \begin{pmatrix}
    c_{12}c_{13} & s_{12}s_{13} & -s_{12}s_{13} \\
    s_{12}c_{13} - c_{12}s_{13}s_{23}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & c_{13}s_{23} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{13}c_{23}
\end{pmatrix}. \quad (2.16)
$$

Here $c_{ij}$ and $s_{ij}$ are cosines and sines of the respective angle $\theta_{ij}$ and $\theta_{12}$ is the Cabibbo angle.

The Wolfenstein parametrization expands the matrix elements in powers of the $\lambda < 1$ parameter, which is possible due to their hierarchy. This way the suppression of transitions between the quark families can be easily seen. In the Wolfenstein parametrization, to the order $\lambda^3$, the CKM matrix becomes

$$
V = \begin{pmatrix}
    1 - \lambda^2/2 & \lambda & \lambda^3(\rho - i\eta) \\
    -\lambda & 1 - \lambda^2/2 & \lambda^2 \\
    \lambda^3(1 - \rho - i\eta) & -\lambda^2 & 1
\end{pmatrix}. \quad (2.17)
$$

Here $\lambda$ equals to the sine of Cabibbo angle, and CP-violation is encoded in $\rho - i\eta$. 
2.3.2 Geometrical Representation

With Eq. 2.14 one can construct six triangles on a complex plane, all of them having the same area \( A = \frac{1}{2} J_{CP} \), where \( J_{CP} \) is the Jarlskog invariant [26]. The triangles containing the scalar products of neighboring rows or columns are almost degenerate. The most commonly used unitarity triangle is represented by equation

\[
V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0. \tag{2.18}
\]

This triangle is shown on Figure 2.3. Its properties are accessible by studying B-meson decays and oscillations. Its sides have comparable length and its angles are defined as

\[
\alpha = \text{arg}(-V_{td} V_{tb}^*/V_{ud} V_{ub}^*) \]
\[
\beta = \text{arg}(-V_{cd} V_{cb}^*/V_{td} V_{tb}^*) \tag{2.19}
\]
\[
\gamma = \text{arg}(-V_{ud} V_{ub}^*/V_{cd} V_{cb}^*)
\]

Currently, the measurements of the CKM triangle show no evidence of the unitarity violation (see Figure 2.4.)

2.4 Effective Field Theory

In physics in general, phenomena of interest can occur at different regions of the parameter space. In particle physics, this parameter space is defined by the typical energy of the interactions. Often it is not possible to construct a theory that would describe the phenomena at all energy scales at once, so the typical approach is to use an Effective Field Theory (EFT) [28]. The basic idea of an EFT is to neglect any effects of the physics above the energy scale of interest and only explain the “effective” physics
below it. The ignored effects are small and can be calculated as perturbations. The EFT approach offers reliable results when the energy scale of interest and the energy scale of the underlying dynamics are of different order. For instance, in the loop diagrams, if $k$ is the loop momentum, the perturbative expansion in terms of $k^2/M_W^2$ will only work if $k^2 \ll M_W^2$.

Typical energy scales are QCD scale $\Lambda_{\text{QCD}} \approx 200 \text{ MeV}$, hadronization scale $\Lambda_{\text{had}} \approx 1 \text{ GeV}$ and electroweak scale $\Lambda_{\text{EW}} \approx 80 \text{ GeV}$. Although the choice of the separation scale is arbitrary, when constructing the EFT, the couplings in the effective theory must be matched to those of the full theory at the chosen scale.

The neglect of high energy effects corresponds to replacing the nonlocal interactions from virtual heavy particle exchange with a set of local interactions, constructed to give the same physics at low energies. The best known example of an EFT in particle physics is Fermi’s current-current interaction, proposed by Enrico Fermi in 1933 to explain the beta decay \cite{Fermi1934, Fermi1935}. Typical reactions studied there are

\begin{align}
n \rightarrow p + e^- + \bar{\nu}_e,
\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu.
\end{align}
2.4. Effective Field Theory

Fermi’s theory assumed a point-like interaction between the four fermions involved in this kind of reactions (see Figure 2.5, left). Due to the large mass of the intermediate $W^-$ boson (Figure 2.5, right), the range of the interaction is short and the point approximation is valid. Similarly to that, in B-physics the effects of a short-ranged force mediated by a heavy boson can be expressed with a point-like interaction.

2.4.1 Operator Product Expansion

An algebraic tool that allows to separate short- from long distance effects is the Operator Product Expansion (OPE). It is a series expansion of the product of two local operators and can be written as

$$O_1(x)O_2(y) = \sum_i C_i(x-y)O_i(y),$$

where the sum runs over a finite number of terms.

In particle physics, following the idea of EFT, one wants to represent the amplitude of a process as a sum of terms with factorized short- and long distance effects:

$$A = \langle H_{\text{eff}} \rangle = \sum_i C_i(\mu) \langle Q_i(\mu) \rangle.$$  (2.22)

Here $C_i$ are Wilson coefficients [31] that contain the short distance effects above the scale $\mu$ and $\langle Q_i(\mu) \rangle$ are the operator matrix elements describing the long distance behavior.

For instance, the amplitude of a decay of a meson $M$ into a final state $F$ is given by [32]

$$A(M \to F) = \langle F | H_{\text{eff}} | M \rangle = \frac{G_F}{\sqrt{2}} \sum_i V_{\text{CKM}} C_i(\mu) \langle F | Q_i(\mu) | M \rangle.$$  (2.23)

Coefficients $C_i$ include the contributions due to heavy particles exchange, and therefore depend on their masses. They can be calculated using perturbative methods. The most important source of the theoretical uncertainty comes from the hadronic matrix elements. A number of approaches, such as lattice calculations, the $1/N$ expansion ($N$ is the number of colors), Quantum Chromodynamics (QCD) sum
rules, hadronic sum rules, chiral perturbation theory and others, exist to obtain quantitative estimates of the matrix elements \[33\].

Although both the effective operators and the Wilson coefficients have to be renormalized and therefore depend also the renormalization scheme, in the effective Hamiltonian these dependencies cancel each other. Thus, the physical quantities do not depend on the choice of \(\mu\) or the renormalization scheme. For B decays, \(\mu\) is typically of the order of the \(b\)-quark mass \(m_b = 4.2\) GeV.

### 2.4.2 Effective Field Theory of rare B-Meson Decays

The application of the OPE technique for the case of \(b \rightarrow s l^+ l^-\) transitions is discussed, for example, in \[34\]. The effective Hamiltonian in this case can be written as

\[
H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V^*_{ts} V_{tb} \sum_{i=1}^{10} C_i Q_i. \tag{2.24}
\]

The set of operators \(Q_i\) contains:

- **Current - Current operators**
  
  \[
  Q_1 = (\bar{s}_i c_j)_{V-A}(\bar{c}_j b_i)_{V-A}
  \]
  
  \[
  Q_2 = (\bar{s} c)_{V-A}(\bar{c} b)_{V-A}
  \]

- **QCD penguins operators**
  
  \[
  Q_3 = (\bar{s} b)_{V-A} \sum_q (\bar{q} q)_{V-A}
  \]
  
  \[
  Q_4 = (\bar{s}_i b_j)_{V-A} \sum_{ij} (\bar{q}_j q_i)_{V-A}
  \]
  
  \[
  Q_5 = (\bar{s} b)_{V-A} \sum_q (\bar{q} q)_{V+A}
  \]
  
  \[
  Q_6 = (\bar{s}_i b_j)_{V-A} \sum_{ij} (\bar{q}_j q_i)_{V+A}
  \]

- **Magnetic penguins operators**
  
  \[
  Q_7 = \frac{\alpha}{4\pi m_b} (\bar{s}_i \sigma^{\mu\nu}(1 + \gamma_5)) b_i F_{\mu\nu}
  \]
  
  \[
  Q_8 = \frac{\alpha}{4\pi m_b} (\bar{s}_i \sigma^{\mu\nu}(1 + \gamma_5)) T^a_{ij} b_i G^a_{\mu\nu}
  \]

- **Semileptonic operators**
  
  \[
  Q_9 = (\bar{s} b)_{V-A}(\bar{\ell} l)_{V}
  \]
  
  \[
  Q_{10} = (\bar{s} b)_{V-A}(\bar{\ell} l)_{A}
  \]

Here \(l\) and \(q\) denote a lepton and a quark, respectively, \(c, s, b\) denote the corresponding quarks and index \(i, j\) stands for the quark color charge. The subscripts \(V, A, V - A\) and \(V + A\) stand for vector, axial, vector-axial and vector+axial, respectively. Then, \(\sigma^{\mu\nu}\) is the commutator of the \(\gamma\)-matrices, \(T^a_{ij}\) is the
generator of the SU(3) group and $G_{\mu\nu}^a$ is the gluonic field strength tensor. The corresponding Feynman diagrams are shown on Figure 2.6.

In order to separate the potential contributions of BSM physics, Eq. 2.24 is often written as

$$H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1} C_i Q_i + C'_i Q'_i,$$  \hspace{1cm} (2.25)

where the primed operators correspond to the BSM processes. Some BSM scenarios will be discussed in Section 2.10.

### 2.5 Physics at the LHC

At the LHC protons can collide at a centre of mass energy of $\sqrt{s} = 8$ TeV (reached in the year 2012). Scattering processes there can be classified as either hard (with high transverse momentum $p_T > 2$ GeV, roughly) or soft. Most of the collisions are soft, only about 1% of the events involve the hard interactions.
CHAPTER 2. THEORETICAL BACKGROUND

Figure 2.7: Parton Density Functions in a proton [36].

The hard scattering processes at the LHC are discussed in [35]. The rates and event properties of the hard process can be calculated using perturbation theory with a good precision. Perturbative calculations, however, are not effective for describing the soft processes, which remain less well understood. At high energy, the proton-proton interaction proceeds via the partonic constituents: quarks and gluons. Not only the valence quarks play a role here, but also the sea quarks and gluons. The proton structure can be presented with a Parton Density Function (PDF) \( f_i(x, Q^2) \), where \( i \) is the index of a parton and the Bjorken scaling variable \( x \) is the fraction of the proton momentum carried by a single parton. PDFs are the parametrization of the partonic content, these functions are derived from the fits to the data collected in Deep Inelastic Scattering (DIS) experiments. In Figure 2.7 a proton PDF for two different energies \( Q^2 = 1 \text{ GeV}^2 \) and \( Q^2 = 10^4 \text{ GeV}^2 \) is presented. The distributions of \( u \)- and \( d \)- quarks are higher than those of their antiparticles due to the presence of valence quarks; the gluon is downscaled by a factor of 10.

The proton-proton interaction cannot be described as an interaction of the valence quarks, it is much more complicated, with the rates of various interaction processes depending on the PDF. A typical collision event is schematically shown in Figure 2.8. The hard process (HP) with a large energy transfer between the two partons is usually of the main interest, but it is always accompanied by a number of soft interactions, which are summarized together as the underlying event (UE). The outgoing particles of the
Figure 2.8: Scheme of a collision event: a) gluon emission and absorption, b) $q\bar{q}$-pair production and annihilation, c) gluon-gluon interaction.

Figure 2.9: $b$-quark production mechanisms.

underlying event are mainly going into the large pseudorapidity (see Eq. 3.3) region (forward direction of the detector), while those originating from a hard process have large transverse momenta, which correspond to the small pseudorapidity. After losing the energy by the emission of radiation partons recombine to color singlet states. This process is called hadronization and the hadrons containing a $b(\bar{b})$-quark are called B-hadrons.

2.5.1 Beauty Production

According to [37], “three mechanisms contribute to the beauty production at hadron colliders: gluon-gluon fusion and $q\bar{q}$-annihilation (flavor creation in hard QCD scattering), flavor excitation (semi-hard
process) and gluon splitting (soft process). Flavor creation refers to the lowest order, two-to-two QCD $b\bar{b}$-production diagrams. Flavor excitation corresponds to the diagrams where a $b\bar{b}$-pair from the quark sea of the proton is excited into the final state when one of the $b$-quarks undergoes a hard QCD interaction with a parton from another proton. In the third type of production processes, the $b\bar{b}$-pair arises from a $g \rightarrow b\bar{b}$ splitting in the initial or final state. The corresponding Feynman graphs are shown in Figure 2.9. Gluon fusion is the dominant $b\bar{b}$-production process at the LHC.

According to QCD factorization theorem [38], the $b\bar{b}$ production cross-section can be calculated as

$$\sigma_{b\bar{b}} = \sum_{i,j} \int \int dx_1 dx_2 f_i(x_1, \mu^2) f_j(x_2, \mu^2) \hat{\sigma}_{i,j},$$

(2.26)

where $\hat{\sigma}_{i,j}$ is the cross section of each individual process and $f_{i/j}$ are the PDFs of the corresponding parton. Theoretical prediction for $b$-quark production cross-section at the LHC is shown in Figure 2.10. The cross section for production of $b\bar{b}$-pairs at $\sqrt{s} = 7$ TeV (14 TeV) is about 300 $\mu$b (500 $\mu$b). Inclusive $b\bar{b}$ cross section measurements by the four LHC experiments show agreement with the theoretical predictions. The studied channels include $J/\psi + X$ final state at LHCb [39] and ALICE [40] experiments, partially reconstructed $D^* \mu^- + X$ final state at ATLAS [41] and $\mu^- \mu^+ X$ at CMS [42]. Indirect production of $b \rightarrow J/\psi + X$ at different proton collision energies measured by LHCb experiment is shown in Figure 2.11.

2.5.2 B-Physics Prospects

The experimental interest in B-physics is motivated by its ability to provide information about physics at very short distances due to the large mass of $b$-quark. The main topics of research in B-physics include CP-violation and rare flavor changing processes. Rare decays that involve FCNC occur only at the loop level in the SM, so studying such processes can provide information on masses and couplings of the virtual particles in the loops, including those predicted in various BSM scenarios. Such indirect searches are a unique opportunity for the detection of BSM effects, since the direct production of such heavy particles may be impossible with the currently available energies.

In particular, FCNC involving $b \rightarrow s \ell^+ \ell^-$ transitions are sensitive to the potential BSM contributions that are contained in the Wilson coefficients $C_7$, $C_9$ and $C_{10}$ (see Eq. 2.25). In Figure 2.12 the diagrams of $b \rightarrow s \ell\ell$ decays in the SM (a), SUSY (b) and a two Higgs-doublet model (c) are shown. The presence of additional particles in the loop can alter the observed angular distributions of the decay products. In this thesis, the exclusive decay $B^0_d \rightarrow K^{*0} \mu^+ \mu^-$ was used to study $b \rightarrow s \ell\ell$ transitions.
Figure 2.10: Standard Model cross sections at the LHC and Tevatron [35] (plot updated by J.Stirling in 2012).
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2.6 Introduction to the $B^0_d \rightarrow K^{*0} \mu^+ \mu^-$ Decay

The $B^0_d \rightarrow K^{*0} \mu^+ \mu^-$ process is a semileptonic rare decay with the branching ratio $\mathcal{B}(B^0_d \rightarrow K^{*0} \mu^+ \mu^-) = (1.06 \pm 0.1) \cdot 10^{-6}$ [16]. It the final state, four tracks are observed, since $K^{*0}$-meson has a very short lifetime ($\tau \sim 10^{-20}$ s) and decays into a kaon and a pion in almost 100% of cases [16]. The charges of these hadrons are defined by the strangeness conservation (so the $K^- \pi^+$ combination is forbidden) and the values of the corresponding Clebsch-Gordan coefficients, given in [16]. This results in the following
2.6. INTRODUCTION TO THE $B^0_d \rightarrow K^{*0} \mu^+ \mu^-$ DECAY

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.13}
\caption{Lowest order Feynman diagrams of the $B^0_d \rightarrow K^{*0} \mu^+ \mu^-$ decay.}
\end{figure}

<table>
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<th>Particle</th>
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<th>Angular momentum</th>
<th>Isospin $I_3$</th>
</tr>
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<td>0</td>
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</tr>
<tr>
<td>$K^{*0}$</td>
<td>$\pi d$</td>
<td>895.94</td>
<td>1</td>
<td>-1/2</td>
</tr>
<tr>
<td>$K^+$</td>
<td>$\pi u$</td>
<td>493.68</td>
<td>0</td>
<td>1/2</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td>$\bar{\pi} d$</td>
<td>139.57</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>$\mu^\pm$</td>
<td>-</td>
<td>105.66</td>
<td>1/2</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2.2: Properties of the particles involved in the $B^0_d \rightarrow K^{*0} \mu^+ \mu^-$ decay [16].

The observables in the $B^0_d \rightarrow K^{*0} \mu^+ \mu^-$ decay are usually considered as functions of the invariant mass of di-muon system $q^2$. The lowest $q^2_{\text{min}}$ and the highest $q^2_{\text{max}}$ possible values of $q^2$ correspond to the cases when either the di-muon pair or the $K^{*0}$ meson are produced at a rest:

$$q^2_{\text{min}} = (m_{\mu^+} + m_{\mu^-})^2 = 0.045 \text{ GeV}^2,$$
$$q^2_{\text{max}} = (m_{B^0_d} - m_{K^{*0}})^2 = 19.22 \text{ GeV}^2.$$ 

The probabilities of the $K^{*0}$ decay final state:

$$K^{*0} \rightarrow K^+ \pi^-,$$ \quad \(~66.6\%), \quad (2.27)
$$K^{*0} \rightarrow K^0 \pi^0,$$ \quad \(~33.3\%).

In this analysis, only the $K^+ \pi^-$ final state is considered. Lowest order Feynman diagrams of $B^0_d \rightarrow K^{*0}(\rightarrow K^+ \pi^-) \mu^+ \mu^-$ decay are presented in Figure 2.13. In Table 2.2 some properties of the particles involved in this decay are summarized.
2.7 The $B^0_d \to K^{*0}\mu^+\mu^-$ Decay Geometry and Angular Distributions

The $B^0_d \to K^{*0}\mu^+\mu^-$ decay result in four particles in the final state. In general, the kinematics of a four-body state can be defined by five variables. In case of the $B^0_d \to K^{*0}\mu^+\mu^-$ decay following variables are usually chosen: the invariant mass $q^2$ of the di-muon system, the invariant mass $p^2$ of the $(K\pi)$-system and three angles describing the geometrical configuration of the final state. There are several conventions on the definition of these angles. In the analysis described in Chapter 5 of this thesis, the definition shown in Figure 2.14 is used. Here $\theta_L$ is the angle between the $\mu^+$ and the direction opposite to the $B^0_d$ in the di-muon rest frame, $\theta_K$ is the angle between the $K^+$ and the direction opposite to the $B^0_d$ in the $K^{*0}$ rest frame, and $\phi$ is the angle between the plane defined by the two muons and the plane defined by the kaon-pion system in the $B^0_d$ rest frame.

In the experiment, in the case of the CP-conjugate decay the angles $\theta_L$ and $\theta_K$ would be defined with respect to the tracks of the negatively charged muon and kaon, respectively. In the theoretical convention, $B^0_d$ and $\bar{B}^0_d$ decays are not distinguished.

The dependence of the differential decay rate on the $p^2$ reflects the presence of the S-wave $B^0_d \to K^{*0}\mu^+\mu^-$. Here $K^{*0}$ denotes the spin-0 unpolarized state of the $(K\pi)$-system, called the S-wave, while the spin-1 $K^{*0}$ is called P-wave. Due to interference between S- and P-waves, the angular distributions of the final state particles differ from the case of the pure P-wave state. The S-wave effects are often neglected, and only the decay via vector state, $B^0_d \to K^{*0}\mu^+\mu^-$, is considered. In this case, following [43],....
the differential decay rate can be written as

\[ \frac{8\pi}{3} \frac{d^4\Gamma}{dq^2 d\cos\theta_1 d\cos\theta_K d\phi} = J_1^s \sin^2\theta_K + J_5^c \cos^2\theta_K + (J_2^c \sin^2\theta_K + J_6^c \cos^2\theta_K) \cos 2\theta_t \]

(2.29)

where the coefficients \( J_1 \) depend on the di-lepton invariant mass \( q^2 \) and, generally, on the lepton mass \( m_i \), although the latter dependence is often neglected, since \( m_i^2 \ll q^2 \). The indices \( s \) and \( c \) in Eq. 2.29 refer to the sine and cosine of the angle multiplied by the corresponding \( J_1 \) coefficient. For the CP-conjugated decay, the differential decay rate is obtained from Eq. 2.29 by replacing [43]

\[ J_{1,2,3,4,7}^{(a)} \rightarrow J_{1,2,3,4,7}^{(a)} \quad \text{and} \quad J_{5,6,8,9}^{(a)} \rightarrow -J_{5,6,8,9}^{(a)}. \]

(2.30)

No additional angular dependence can arise from any BSM extension, as shown in [44] and [45].

It must be also mentioned, that in the experiment only the averaged over a certain range of \( q^2 \) values of angular coefficients can be measured. From the theoretical point of view, this corresponds to calculating

\[ \langle J_1 \rangle = \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} J_1(q^2) dq^2 \]

(2.31)

and

\[ \langle \Gamma \rangle = \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} d\Gamma dq^2 \left[ \int_{-1}^{1} d\cos\theta_1 \int_{-1}^{1} d\cos\theta_K \int_{0}^{2\pi} d\phi \frac{d^4\Gamma}{dq^2 d\cos\theta_1 d\cos\theta_K d\phi} \right] \]

(2.32)

and considering the following differential decay rate:

\[ \frac{1}{\langle \Gamma \rangle} \frac{d^3\langle \Gamma \rangle}{d\cos\theta_1 d\cos\theta_K d\phi} = \frac{1}{q_{\text{max}}^2} \int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 \frac{d^4\Gamma}{dq^2 d\cos\theta_1 d\cos\theta_K d\phi} = \]

(2.33)

\[ \frac{3}{8\pi} \frac{1}{\langle \Gamma \rangle} \left[ \langle J_1^s \rangle \sin^2\theta_K + \langle J_5^c \rangle \cos^2\theta_K + (\langle J_2^c \rangle \sin^2\theta_K + \langle J_6^c \rangle \cos^2\theta_K) \cos 2\theta_t \right] \]

+ \langle J_3 \rangle \sin^2\theta_K \sin^2\theta_1 \cos 2\phi + \langle J_4 \rangle \sin 2\theta_K \sin 2\theta_1 \cos \phi \\

+ \langle J_5 \rangle \sin 2\theta_K \sin 2\theta_1 \cos \phi + (\langle J_6 \rangle \sin^2\theta_K + \langle J_7 \rangle \cos^2\theta_K) \cos 2\theta_t \\

+ \langle J_7 \rangle \sin 2\theta_K \sin \theta_1 \sin \phi + \langle J_8 \rangle \sin 2\theta_K \sin 2\theta_1 \sin \phi \\

+ \langle J_8 \rangle \sin^2\theta_K \sin^2\theta_1 \sin 2\phi \right]. \]
In the experiment, the size of the data sample is often too small to perform a fit to the full differential decay rate. In this case, Eq. 2.33 can be integrated over one or two angles. By integrating out the $\phi$ angle one obtains a two-dimensional distribution:

$$
\frac{1}{\langle \Gamma \rangle} \frac{d^2(\Gamma)}{d\cos \theta_1 d\cos \theta_K} = \frac{3}{4} \frac{1}{\langle \Gamma \rangle} \left[ \langle J_1^a \rangle - \langle J_2^a \rangle \right] \cos \theta_K + \sum_{l=1}^{2} \left( \langle J_1^l \rangle \cos \theta_K + \sum_{m=1}^{2} \langle J_2^m \rangle \cos \theta_1 \cos \theta_K \right)
$$

(2.34)

2.8 Observables in the $B^0_d \to K^{*0} \mu^+ \mu^-$ Decay

The angular analysis of the four-body final states of $B \to (K\pi)l\bar{l}$ decays offers a large number of observables to study. Different polarization states of the $K^{*0}$ and chiralities of the di-muon system result in seven complex decay amplitudes $A_m$, often called transversity amplitudes. Coefficients $J_i$ in Eq. 2.29 can be represented as bilinear combinations of these amplitudes, containing terms with $|A_m|^2$, $\Re(A_m A_n^*)$ and $\Im(A_m A_n^*)$. The transversity amplitudes are discussed in [43, 46–48], where it is shown that in the large $q^2$ region they factorize into universal coefficients $C_L, R$ and form-factors $f_m$:

$$
A_L^m, R^m \propto C_L f_m, R f_m, m = \perp, \parallel, 0.
$$

(2.35)

Here L, R refer to the chirality of the lepton current and index $m$ denotes the polarization states. The exact expression of $A_m$ in terms of Wilson coefficients $C$ and QCD form-factors can be found in [48] and [49]. The expression of the angular coefficients $J_i$ through the transversity amplitudes is given in Appendix A, both in the SM as well as under the presence of BSM contributions.

Normalized CP-averages $S_i$ and CP-asymmetries $A_i$ can be defined as [44]

$$
S_i^{(a)} = \frac{J_i^{(a)} + J_i^{(-a)}}{d(\Gamma + \bar{\Gamma})/dq^2}
$$

$$
A_i^{(a)} = \frac{J_i^{(a)} - J_i^{(-a)}}{d(\Gamma + \bar{\Gamma})/dq^2}.
$$

(2.36)

The observables $S_{7,8,9}$ depend on combinations of transversity amplitudes $\Im(A_m A_n^*)$. They are expected to be close to zero over the full range of $q^2$ in the SM as well as in most of BSM scenarios, since they are suppressed by the small size of the strong phase difference between the decay amplitudes. The CP-symmetric observables $A_{7,8,9}$, however, remain sensitive to the potential BSM effects. Detailed analyses of CP-averaged and CP-asymmetric observables can be found in [44, 45, 49].

The first experiments to study the angular distributions of $B \to K^{*1}l^+l^-$ decays were BaBar [8] and Belle [7], which measured two observables - forward-backward asymmetry of the leptons $A_{FB}$ and the
fraction of $K^{*0}$ longitudinal polarization $F_L$. These observables can be expressed in terms of $J_i$ coefficients as \[ [48] \]

\[
\langle A_{FB} \rangle = \frac{1}{\langle I \rangle} \left( \langle J_6^c \rangle + \frac{\langle J_6^c \rangle}{2} \right) \tag{2.37}
\]

\[
\langle F_L \rangle = \frac{1}{\langle I \rangle} \left( \langle J_1^c \rangle - \frac{1}{3} \langle J_2^c \rangle \right).
\]

They correspond to CP-averaged parameters $S_i$ as

\[
A_{FB} = -\frac{3}{8} (2S_6^c + S_6^c) \tag{2.38}
\]

\[
F_L = -S_2^c,
\]

if $m_2^2 \ll q^2$. The advantage of $A_{FB}$ and $F_L$ is their experimental accessibility, they can be extracted from the reduced angular distributions where one or two angles are integrated out, and thus can be obtained from the fit to a small sample of events available. In terms of $A_{FB}$ and $F_L$, Eq. 2.34 can be written as

\[
\frac{1}{\langle I \rangle} \frac{d^2\langle \Gamma \rangle}{d\cos\theta_l d\cos\theta_K} = \frac{9}{16} \left[ \langle F_L \rangle \cos^2\theta_K + \frac{3}{4} (1 - \langle F_L \rangle)(1 - \cos^2\theta_K) 
- \langle F_L \rangle \cos^2\theta_K (2\cos^2\theta_l - 1)
+ \frac{1}{4} (1 - \langle F_L \rangle)(1 - \cos^2\theta_K) (2\cos^2\theta_l - 1)
+ \frac{4}{3} \langle A_{FB} \rangle \cos\theta_l (1 - \cos^2\theta_K) \right].
\] \tag{2.39}

By integrating out one of the $\theta_{K,1}$ angles, one-dimensional distributions can be obtained:

\[
\frac{1}{\langle I \rangle} \frac{d\langle \Gamma \rangle}{d\cos\theta_l} = \frac{3}{4} \langle F_L \rangle (1 - \cos^2\theta_l) \tag{2.40}
\]

\[
+ \frac{3}{8} (1 - \langle F_L \rangle) (1 + \cos^2\theta_l) + \langle A_{FB} \rangle \cos\theta_l,
\]

\[
\frac{1}{\langle I \rangle} \frac{d\langle \Gamma \rangle}{d\cos\theta_K} = \frac{3}{2} \langle F_L \rangle \cos^2\theta_K + \frac{3}{4} (1 - \langle F_L \rangle) (1 - \cos^2\theta_K). \tag{2.41}
\]

The last two equations are only valid if the lepton mass is neglected \[48\].

### 2.9 S-Wave Contribution

In the expressions of various observables in the previous sections, the presence of S-wave (decay of $B \to K^{*0}(\to K^+\pi^-)\mu^+\mu^-$ via an intermediate scalar state $K^{*0}$) was neglected. Taking it into account leads to the dependence of the angular coefficients $J_i$ on the invariant mass of $(K\pi)$-system (see \[50\]).
The complete angular distribution with the S-wave component, calculated in [51], is
\[
\frac{1}{16\pi} \frac{d^5\Gamma}{d^4q^2 d\cos\theta_K d\cos\theta_1 d\phi} = \frac{9}{16\pi} \left( \frac{2}{3} F_S + \frac{4}{3} A_S \cos\theta_K \right) (1 - \cos^2\theta_1)
\]
\[
+ \left( 1 - F_S \right) \left[ 2 F_L \cos^2\theta_K (1 - \cos^2\theta_1) + \frac{1}{2} (1 - F_L) (1 - \cos^2\theta_K)(1 + \cos^2\theta_1) \right.
\]
\[
+ \frac{1}{2} (1 - F_L) A_L^2 (1 - \cos^2\theta_K)(1 - \cos^2\theta_1) \cos 2\phi \right.
\]
\[
+ \frac{4}{3} A_{FB} (1 - \cos^2\theta_K) \cos\theta_1 + A_{im} (1 - \cos^2\theta_K)(1 - \cos^2\theta_1) \sin 2\phi \right) 
\]
where
\[
F_S = \frac{|A_{00}|^2}{|A_{10}|^2 + |A_{11}|^2 + |A_{1\perp}|^2 + |A_{00}|^2} 
\]
\[
A_S = \frac{\sqrt{3}}{2} \frac{|A_{00}| |A_{10}^*| \cos \delta_L + (L \rightarrow R)}{|A_{10}|^2 + |A_{11}|^2 + |A_{1\perp}|^2 + |A_{00}|^2} 
\]
\[
A_{FB} = \frac{3}{2} \frac{\text{Re}(A_{10}^* A_{1\perp}^*) - \text{Re}(A_{10} R_{1\perp}^*)}{|A_{10}|^2 + |A_{11}|^2 + |A_{1\perp}|^2} 
\]
\[
F_L = \frac{|A_{10}|^2}{|A_{10}|^2 + |A_{11}|^2 + |A_{1\perp}|^2} 
\]
\[
A_L^2 = \frac{|A_{1\perp}|^2 - |A_{11}|^2}{|A_{1\perp}|^2 + |A_{11}|^2} 
\]
\[
A_{im} = \frac{\text{Im}(A_{11}^* A_{1\perp})}{|A_{10}|^2 + |A_{11}|^2 + |A_{1\perp}|^2}. 
\]

Here the first lower index \(0,1\) of the transversity amplitude \(A_{mn}\) reflects the spin of the intermediate \(K^0\) meson and corresponds to S- and P-wave, respectively. In Eq. 2.43, \(\delta_L\) is the phase difference between S- and P-wave propagators. In the experiment the extraction of all the parameters from the fit is very challenging as it requires high statistics of the sample and a good understanding of the background sources.

In Figure 2.15 the distributions of \(\cos\theta_1\), \(\cos\theta_K\) and \(\phi'\) are shown under the assumptions of no S-wave and a 7% of S-wave contribution to the total number of events. Here \(\phi'\) is defined as
\[
\phi' = \begin{cases} 
\phi, & \phi \geq 0 \\
2\pi - \phi, & \phi < 0 
\end{cases} 
\]

The S-wave presence results in a decrease of the forward-backward asymmetry \(A_{FB}\) and in the appearance of an asymmetry in \(\cos\theta_K\) distribution. The bias in the measured value of the angular observables \(A_{FB}, F_L, A_L^2\) and \(A_{im}\) is studied in [51] as a function of the data sample size and the S-wave fraction. It is shown that the S-wave effect on the angular observables must be taken into account for datasets with 200 or more signal events, if the S-wave fraction is of the order of 8%, as was estimated by BaBar [52].
2.10. POTENTIAL BSM EFFECTS

Figure 2.15: Distributions of $\cos \theta_l$ (a), $\cos \theta_K$ (b) and $\phi'$ (c) with (dashed blue) and without (solid red) S-wave component of 7% [51].

2.10 Potential BSM Effects

In the scenarios beyond the SM, additional operators enter Eq. 2.24. These are [48]:

- chirality-flipped operators

  \[ Q_{7'} = \frac{m_b}{e} [\bar{s} \sigma^{\mu \nu} P_L b] F_{\mu \nu} \]  
  \[ Q_{9'} = [\bar{s} \gamma_\mu P_R b][\bar{t} \gamma^\mu t] \]  
  \[ Q_{10'} = [\bar{s} \gamma_\mu P_R b][\bar{t} \gamma^\mu \gamma_5 t] \]  

- scalar and pseudo-scalar operators, including chirality-flipped ones

  \[ Q_{S(S')} = [\bar{s} P_{R(L)} b][\bar{t} t] \]  
  \[ Q_{P(P')} = [\bar{s} P_{R(L)} b][\bar{t} \gamma_5 t] \]
• tensor operators

\[ Q_T = [\bar{s}\sigma_{\mu\nu}b][\bar{l}\sigma^{\mu\nu}l] \] (2.47)

\[ Q_{T5} = \frac{i}{2} \epsilon^{\mu\nu\alpha\beta}[\bar{s}\sigma_{\mu\nu}b][\bar{l}\sigma_{\mu\nu}b][\bar{l}\sigma_{\alpha\beta}l] \] (2.48)

In the above, \( p_L = 1 - \gamma_5 \) and \( p_R = 1 + \gamma_5 \) denote left- and right-handed currents, respectively.

The expressions of the angular coefficients in terms of transversity amplitudes for this case are given in Appendix A.

Although both the LHCb [12] and CMS [11] experiments claim to see no deviation from the SM prediction in the measured values of angular observables and \( B_0^d \rightarrow K^{*0}\mu^+\mu^- \) branching fraction, CDF [10] reported a cumulative difference in the \( A_{FB} \) spectrum of 2.7 standard deviation, compared to the SM prediction. In [53] it is shown that the currently available measurements of \( B_0^d \rightarrow K^{*0}\mu^+\mu^- \) and \( B_s^0 \rightarrow \phi\mu^+\mu^- \) decays, specially their branching fractions, might indicate a presence of BSM physics. Various observables in these decays provide the information on the value of Wilson coefficient \( C'_9 \), and the best fitted value corresponds to

\[ C_9 = C'^9_{SM} - 1.1 \] (2.49)

\[ C'_9 = 1.1. \]

In Figure 2.16 the experimental measurements of the forward-backward asymmetry of the muons \( A_{FB} \), fraction of longitudinal polarization of \( K^{*0} \) and the branching fraction of \( B_0^d \rightarrow K^{*0}\mu^+\mu^- \) decay are compared with the SM prediction. A fit with "new physics" model, corresponding to the values of \( C'^9_9 \) coefficients presented in Eq. 2.48, is shown. The experimental result is obtained by averaging the values measured by ATLAS, CMS, LHCb and CDF experiments. In [54], various BSM models are considered that might lead to the observed values of the angular parameters. These models include flavor-changing neutral gauge boson (\( Z' \)), Minimal Supersymmetric Standard Model (MSSM) and models with partial compositeness. The authors conclude that the best fit to the experimental data can be obtained with the following values of "new physics" part of the Wilson coefficients \( C'^9_9 \):

\[ C_9 = C'^9_{SM} - 1.0 \pm 0.3 \] (2.49)

\[ C'_9 = 1.0 \pm 0.5, \]

which agrees with the values in Eq. 2.48.

A model-independent analysis of the experimental results is also presented in [55]. The authors base their calculations on the measurements by LHCb experiment. The obtained values of BSM contributions
2.10. POTENTIAL BSM EFFECTS

Figure 2.16: Averaged experimental values of $A_{FB}$ (a), $F_L$ (b) and $B^0_d \rightarrow K^{*0} \mu^+ \mu^-$ branching fraction (c). "New physics" fit (dashed line) and SM prediction (shaded area) [53].

are

$$-1.6 < C_9 < -0.9$$

$$-0.2 < C_9' < 0.2$$

at 68% confidence level.

Improved results of the measurements of $B^0_d \rightarrow K^{*0} \mu^+ \mu^-$ decay, as well as other similar decays, are expected from the analysis of the data collected in the year 2012 by the three LHC experiments, ATLAS, CMS and LHCb. These more precise results would be necessary to confirm the presence of BSM effects.
Chapter 3

The **LHC** and the **ATLAS** Experiment

In the following chapter, Section 3.1, the overview of the LHC, its parameters and experiments is given. The ATLAS detector is described in Section 3.2, with the details on its geometry and main components: magnetic system, Inner Detector, calorimeter and Muon Spectrometer. The overview of the trigger system is presented in Section 3.3, data processing and storage are discussed in Section 3.4.

3.1 Large Hadron Collider

LHC is a proton-proton collider located in Geneva, Switzerland. It lies in a 27 km long tunnel, 175 m deep under ground, mainly under french territory. The LHC started operating in autumn 2009 colliding protons at the centre-of-mass energies of $\sqrt{s} = 0.9$ TeV and 2.36 TeV, with long periods of collisions at 7 TeV and 8 TeV continuing until February 2013. Currently LHC is going through an upgrade process and collisions will resume in 2015 at the energy of 13 TeV, with a planned raise up to the design energy of 14 TeV.

Before entering the main acceleration ring, the protons have to pass several pre-acceleration steps (see Figure 3.1). The first system is the linear particle accelerator LINAC-2 generating protons with an energy of 50 MeV, that are fed into the Proton Synchrotron Booster (PSB). There the protons get accelerated to 1.4 GeV and further are injected into proton synchrotron, where they are accelerated to 26 GeV. In the last step before the injection into the main LHC ring their energy is increased to 450 GeV by the Super Proton Synchrotron (SPS). The final acceleration to the energy of 8 TeV inside the largest LHC ring is done by a system of eight superconducting cavities.

There are four intersection points of the proton beams, corresponding to the four major experiments at the LHC: ALICE, ATLAS, CMS and LHCb. ATLAS and CMS are the two general purpose detectors with rich physics program, including the study of CP-violation, top quark properties, search for Beyond
the Standard Model (BSM) physics, and one of the main goals, which already resulted in great success - the discovery of the Higgs boson. Large Hadron Collider beauty (LHCb) is a dedicated B-physics experiment, with the main goal of measuring the parameters of CP-violation in the interactions of b-hadrons. A Large Ion Collider (ALICE) is constructed to study heavy ion collisions and explore the quark-gluon plasma.

The number of events per second generated in the LHC collisions is given by

\[ N_{ev} = \sigma L, \quad (3.1) \]

where \( \sigma \) is the cross section of the process and \( L \) is the machine luminosity. The luminosity depends on the beam parameters and can be written as

\[ L = \frac{N^2nf}{4\pi\epsilon_n\beta^* F}, \quad (3.2) \]

where \( N \) is the number of particles per bunch, \( n \) is the number of bunches per beam, \( f \) is the revolution frequency, \( \epsilon_n \) is the normalized transverse beam emittance, \( \beta^* \) is the beta function related to the transverse size of the beam and \( F \) is the geometric luminosity reduction factor depending on the beams crossing angle at the interaction point. The beam parameters may vary between the runs, the actual values set during the LHC operation can be found in [56]. Bending, steering and focusing of the proton beams is controlled by a system of superconducting magnets. The 1232 dipole magnets are responsible for bending, and 392 quadrupole magnets - for focusing of the beams. The magnets are kept at the operating temperature of 1.9 K with 96 tons of superfluid helium-4 used for cooling. More details on the LHC operation can be found in [57] and [58].
3.2 **ATLAS Detector**

**ATLAS** stands for A Toroidal LHC Apparatus. It owes this name to the shape of its large magnets. It is the largest of the **LHC** detectors, 45 m long, 25 m in diameter, and weighs about 7000 tons. The overall view of the detector is presented in Figure 3.2. It consists of three main parts: Inner Detector (ID), calorimeter and Muon Spectrometer (MS), as shown in Figure 3.3. Unlike **LHCb**, **ATLAS** has no dedicated detector for hadron type identification. Detailed information about the construction and performance of the detector can be found in [59].

3.2.1 **Coordinate System**

The origin of **ATLAS** coordinate system is the intersection point of the two proton beams, located at the center of the detector. The axis system is right-handed, the positive \( x \)-axis points to the center of the **LHC** tunnel, \( y \)-axis - to the surface, and \( z \)-axis is collinear to the beam line. Sometimes it is useful to define the position in the cylindrical coordinate system \((R, \theta, \phi)\). In this system the transverse distance to the \( z \)-axis is defined as the radius \( R \). The polar angle \( \theta \in [0, \pi] \) is measured from the positive \( z \)-axis, the azimuthal angle \( \phi \in [-\pi, \pi] \) is measured in the \( xy \)-plane, such that the positive \( x \)-axis corresponds to \( \phi = 0 \) and the positive \( y \)-axis to \( \phi = \pi/2 \). Instead of the polar angle, the pseudorapidity quantity is often used since it is invariant under the Lorentz transformation. Pseudorapidity \( \eta \) is expressed in terms of
the polar angle as
\[ \eta = -\ln \left( \tan \frac{\theta}{2} \right), \] (3.3)
and in terms of momentum components as
\[ \eta = \frac{1}{2} \ln \left( \frac{|p| - p_z}{|p| + p_z} \right) \] (3.4)

From this expression it can be seen, that the pseudorapidity equals to the rapidity \( y \) in the limit of zero particle mass
\[ y = \frac{1}{2} \ln \left( \frac{E - p}{E + p} \right) \] (3.5)

In the \( xy \)-plane \( \eta = 0 \) and along the beam direction \( \eta = \pm \infty \).

### 3.2.2 Magnetic System

Large superconducting magnets are used to bend charged particles in the detector, so their momenta can be measured. The magnetic system of ATLAS consists of four magnets: the central solenoid, the barrel toroid, and two endcap toroids. The central solenoid has a length of 5.8 m and a diameter of 2.5 m and creates an axial symmetric magnetic field of 2 T. Its thickness has to be low in order to decrease the chance of particle interaction with the solenoid material and keep the collision event image clean. The
solenoid is kept at the temperature of 4.5 K, when its material - NbTi - is in the superconductive state. The support cylinder is made of aluminum.

The outer magnetic system (barrel and endcap toroids) provides the configuration of magnetic lines that are orthogonal to the particle trajectories. The barrel toroid creates the magnetic field of 0.5 T and the endcap toroids of 1 T. The distance between the inner and the outer parts of the magnetic system is large enough so that the toroidal field does not influence the solenoidal one.

### 3.2.3 Inner Detector

A detailed information on the ATLAS Inner Detector (ID) can be found in [60] and [61]. The ID is constructed to provide an excellent resolution of the track and vertex position. If one considers B-mesons with an average lifetime of $\tau = 0(10^{-12})$ s, the distance between primary (production) and secondary (decay) vertex in this case is $c\tau = \mathcal{O}(100 \, \mu m)$, which requires a large number of sensitive layers located as close as possible to the interaction point in order to be resolved. On the other hand, the dense layer structure would decrease the tracking precision due to diffraction. Another challenge comes from the high event rate, that puts high demands on the electronics operation speed.

The schematic view of the ID is presented in Figure 3.4. It has a cylindrical shape, with the length $l = 7024$ mm and the radius $R = 1150$ mm and it covers the pseudorapidity range $|\eta| < 2.5$. It consists of three complementary sub-detectors: pixel detector (Section 3.2.3.1), Semiconductor Tracker (SCT) (Section 3.2.3.2) and Transition Radiation Tracker (TRT) (Section 3.2.3.3), each of them separated into a barrel and two endcap regions. The detector resolution depends on the pseudorapidity, with higher resolution achieved in the low pseudorapidity range (barrel region). The detector must operate in the high-radiation environment, which requires the silicon sensors to be kept at low temperature (between $-10^\circ C$ and $-5^\circ C$) so the noise level stays low. The TRT is designed to operate at room temperature.

The magnetic field created by the central solenoid bends the particle trajectory such that the measurement of the transverse momenta down to $p_T = 0.5$ GeV is possible. Particles with lower transverse momentum are bent to very small radii, preventing them from leaving the ID. Additionally, the efficiency of low momentum tracks reconstruction is reduced due to the material effects.

Both the ID and the magnetic system material are required to have low radiation length $X_0$ and nuclear interaction length $\lambda$. The material distribution in the ID is shown in Figure 3.5 in units of $X_0$ (left) and $\lambda$ (right). Different colors correspond to the various detector components.
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Figure 3.4: Schematic view of the Inner Detector.

Figure 3.5: Material distribution in the ID as a function of $\eta$ in units of radiation length $X_0$ (left) and nuclear interaction length $\lambda$ (right) [59].
3.2. **ATLAS DETECTOR**

3.2.3.1 **Pixel Detector**

The part of the ID closest to the collision point is the silicon pixel detector, with three barrel layers located at a distance of 50.5 mm, 88.5 mm and 122.5 mm from the beam line and a length of 800.1 mm. Three endcap disks perpendicular to the beam axis are built at the distances of 495 mm, 580 mm and 650 mm in \( z \)-direction. The pixel detector consists of 1744 Modules, each with 47232 pixels, where 286, 494 and 676 modules are in the three barrel layers respectively, and 48 modules are in each of the six disks. The total number of readout channels is therefore about 80.5 millions.

A pixel module is made of 250 \( \mu \text{m} \) thick \( n \)-type silicon semiconductor. Charged particles traversing the detector create electron-hole pairs, the created charge is then collected in the pixels, allowing to measure the position of the passed particle with an accuracy of 10 \( \mu \text{m} \) in the ‘R-\( \phi \)’ direction and 115 \( \mu \text{m} \) in \( z \)-direction. Due to the radiation damage, the detection efficiency of the layers decreases with time, with the estimated drop of 6% for the innermost layer. It is currently being replaced according to the upgrade plan [62].

3.2.3.2 **Semiconductor Tracker**

The SCT is the middle component of the ID, that consists of four layers in the barrel region and nine in the endcap, located at the radial distances shown in Figure 3.6 and Figure 3.7, respectively.

The position of a particle in the barrel region is measured with a double layer of microstrip sensors, where one layer is arranged at a stereo angle of 40 mrad with respect to the other layer. The microstrips have a pitch width of 80 \( \mu \text{m} \) and a length of 6 cm. The operating principle is similar to that of the pixel detector: electron-hole pairs are created by charged particles and the charges are collected in one end of a strip. The operation voltage is in the order of 250 V to 350 V. The SCT consists of 2112 modules in the barrel region and 1976 modules in the 18 endcap layers. It has 6.3 million of readout channels in total. In the barrel region, the intrinsic measurement accuracy is 17 \( \mu \text{m} \) in the ‘R-\( \phi \)’-direction and 580 \( \mu \text{m} \) in \( z \)-direction.

3.2.3.3 **Transition Radiation Tracker**

The TRT is the outermost layer of the ID, starting from the radius of 554 mm up to 1082 mm. The TRT modules are made of drift tube layers interbedded with fibers in the barrel region and foils on the endcap, which provide transition radiation. The intensity of the transition radiation emitted by the relativistic charged particles when crossing the interface of two materials with different dielectric constants is proportional to their energy and mass. Due to that, electrons and pions can be distinguished with the misidentification probability of a few percents.
The basic detecting elements of the TRT are drift tubes made of polyimide, each 4 mm in diameter and 144 cm long in barrel region, 37 cm long in the endcap. The tubes are filled with 70% of Xe, 27% of CO\textsubscript{2} and 3% of O\textsubscript{2}. The gas becomes ionized when a charged particle passes through, the straw tube works as a cathode, and the anode in the centre of the tube is a 31 \( \mu \)m thick wolfram wire plated with a thin film of gold. The TRT has about 298000 straws in total. The 52544 tubes in the barrel are divided in the middle of each tube, so the wolfram wire is electrically split into two parts that are read out on each side. The straw tubes are arranged parallel to the beam pipe. In the endcap regions, the 245670 straw tubes arranged radial are read out at the outer radius. This gives a total number of readout channels of about 350000.

The intrinsic measurement accuracy for a single straw tube is 130 \( \mu \)m. The TRT pseudorapidity coverage goes up to \(|\eta| < 2.0\) in the endcap region and \(|\eta| < 1.0\) in the barrel region.
3.2. **ATLAS DETECTOR**

The calorimeter is located outside the magnets that surround the ID. Its purpose is to measure the energy of particles, except muons and neutrinos that escape undetected. A particle entering the calorimeter loses all its energy in a particle shower. This energy is reconstructed by measuring ionization, scintillation light or Cherenkov radiation. The calorimeter must be able to measure missing transverse energy $E_{\text{miss}}^T$ with a good resolution, so hermicity is one of the important requirements. There are two basic calorimeter systems: an inner Electromagnetic (EM) calorimeter and an outer hadronic calorimeter. Both are sampling calorimeters; that is, they absorb energy in high-density metal and periodically sample the shape of the resulting particle shower, obtaining the energy of the initial particle from this measurement. Beside that, both calorimeters are separated into barrel and endcap regions, with overall coverage of $|\eta| < 4.9$. The overview of the ATLAS calorimeter is presented in Figure 3.8.

The EM calorimeter is used to measure the energy of electrons, positrons and photons. It is very precise in measuring both the amount and the location of the energy deposited. The EM calorimeter has accordion shaped electrodes, thus providing full coverage in $\phi$ and ensuring a uniform performance in terms of linearity and resolution as a function of $\phi$. The absorbers are made of lead and stainless steel, Liquid Argon (LAr) acts as active material. The energy resolution of the calorimeter depends on the incoming particle energy and was measured to be [59]

$$\frac{\sigma(E)}{E} = 10\% \oplus 0.17\%,$$

(3.6)
where the first number is the value of the stochastic term and the second reflects the non-linearity in the calorimeter response.

The hadronic calorimeter measures the energy of particles that interact via the strong force. It is less precise than the electromagnetic, both in energy magnitude and in the localization, and mainly aims at the reconstruction of jets and the measurement of missing energy. The three main parts of the hadronic calorimeter are: the tile calorimeter, the liquid-argon Hadronic Endcap Calorimeter (HEC) and the liquid-argon Forward Calorimeter (FCal). The tile calorimeter is 8 m in diameter and covers 12 m along the beam axis. The energy-absorbing material is steel, with scintillating tiles that sample the deposited energy. The tiles are oriented radial and perpendicular to the beam line. The iron in the tile calorimeter also acts as a return yoke of the magnetic field lines of the central solenoid. Its energy resolution measured with a test pion beam is [63]

$$\frac{\sigma(E)}{E} = \frac{53\%}{E} \oplus 5.7\%.$$  \hspace{1cm} (3.7)

In the HEC, copper is used as absorber and liquid argon as the active material. HEC has a flat-plate design and covers the range $1.5 < |\eta| < 3.2$. The measured energy resolution is [64]

$$\frac{\sigma(E)}{E} = \frac{70\%}{E} \oplus 5.8\%.$$  \hspace{1cm} (3.8)

The FCal provides coverage over the range of pseudorapidity $3.1 < |\eta| < 4.9$. It has one electromagnetic
and two hadronic layers. Since the FCal modules are located at the large pseudorapidity, 4.7 m far from the interaction point in z-direction, they are exposed to high radiation flux. The FCal energy resolution measured with electron and pion beams, respectively, is \[ \sigma(E_e) = \frac{29\%}{E_e} \oplus 3.5\% \]
\[ \sigma(E_\pi) = \frac{94\%}{E_\pi} \oplus 7.5\%. \] (3.9)

In order to absorb the electromagnetic and hadronic showers and prevent punch-through into MS, the calorimeter depth must be high. In Figure 3.9 the material distribution in the barrel (left) and endcap (right) regions of the EM calorimeter is shown as a function of \( \eta \) in units of radiation length \( X_0 \) and nuclear interaction length \( \lambda \). Figure 3.10 shows the cumulative material distribution in various calorimeter modules.

### 3.2.5 Muon Spectrometer

The Muon Spectrometer (MS) is the outermost part of the ATLAS detector, covering the pseudorapidity range of \( |\eta| < 2.7 \). It is designed to measure the momenta of charged particles exiting the calorimeter. Ideally, all the particles except for muons and neutrinos are stopped inside the calorimeter, so only muons can create hits in the MS. Muons with low transverse momentum \( p_T < 0.5 \text{ GeV} \) cannot reach the MS.
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Figure 3.10: Material distribution in various calorimeter modules as a function of $\eta$ in units of the radiation length $X_0$ [59].

Figure 3.11: Schematic view of the MS.
Various components of the MS are shown in Figure 3.11. Among them, the Monitored Drift Tube (MDT) chambers and Cathode Strip Chambers (CSC) are used for the precision tracking and are described in Section 3.2.5.1, while Resistive Plate Chambers (RPC) and Thin Gap Chambers (TGC) are used for triggering and will be mentioned in Section 3.2.5.2. Detailed information on the construction and performance of the MS is available in [66] and [67].

### 3.2.5.1 Muon Tracking Chambers

Precision-tracking chambers in the barrel region are located between and on the eight coils of the superconducting barrel toroid magnets, while the endcap chambers are in front and behind the two endcap toroid magnets. The chambers in the barrel consist of three layers at radii of 5 m, 7.5 m, and 10 m around the beam axis. In the two endcap regions, muon chambers are perpendicular to the \( z \)-axis and located at distances of \(|z| \approx 7.4 \) m, 10.8 m, 14 m, and 21.5 m from the interaction point. In the pseudorapidity range \(|\eta| < 2.7\) MDT are used for the measurement of muon track coordinates, except in the innermost endcap layer where in the range of \( 2.0 < |\eta| < 2.7\) CSC are used.

MDTs have a diameter of about 30 mm and a wall thickness of 0.4 mm and are filled with a mixture of Ar (93 %) and CO\(_2\) (7 %) at a pressure of 3 bar. Inside the tube a 50 \( \mu \)m thick wolfram-rhenium wire covered with a thin film of gold works as anode at a voltage of 3080 V. Charged particles ionize the gas in the tube, creating electrons. These electrons drift to the central wire, further ionizing the gas and producing a cascade of electrons, which all together are collected in the wire creating a signal. Three or four layers of MDT are arranged in one multilayer, and the MDT chamber is formed by two of such multilayers. A total area of 5500 m\(^2\) is covered by 1088 MDT chambers. Only the coordinate in the bending plane can be measured with the MDT. The resolution of a MDT layer is 80 \( \mu \)m, at the worst, taking into account its degrading due to irradiation.

CSC were chosen to be used in the most forward region of the inner layer due to the high radiation caused by thermalised neutrons from the calorimeter. The CSC are multiwire proportional chambers, providing the ability to measure both \( \eta \) and \( \phi \) coordinates when a muon is passing the CSC. The position resolution of the CSC plane is 60 \( \mu \)m in the bending (\( \eta \)) and about 5 mm in the transverse (\( \phi \)) direction. The CSCs are filled with Ar (80 %) and CO\(_2\) (20 %). This gas mixture provides a low neutron sensitivity, and the small gas volume – a short drift time of less then 40 ns. In total, the timing resolution of a CSC is about 7 ns.

The determination of the muons momenta requires a measurement of their trajectory at three space points. The principle of this measurement differs between the barrel and the endcap region due to the different positioning of the MDT layers: in the barrel region one layer of muon chambers is placed...
within the toroidal magnet system, and in the endcaps the toroid is placed between the first and the second layer. The measurement idea is shown in Figure 3.12. In the barrel region, the muon momentum is calculated by measuring sagitta $s$ (the trajectory deviation from a straight line), and the direct distance $L$ between the two measurement points in the first and third layer. If $B$ is the applied magnetic field, the transverse momentum is calculated as

$$p_T = \frac{L^2 B}{8s}. \quad (3.10)$$

In the endcap region, the trajectory is bent between the first two layers of muon chambers, but is straight between the second and the third layer. The reconstruction of the muon momentum depends on the point angle measurement.

The MS is able to measure the muon momenta from a few GeV up to a TeV. The muon momentum resolution as a function of the transverse momentum is shown in Figure 3.13 for barrel (left) and endcap (right) region.

### 3.2.5.2 Muon Trigger

In addition to the measurement of the muons momenta, the muon system is required to trigger on the passing muons. The tracking chambers have therefore been supplemented with a system of fast trigger chambers that can provide the track information within less than a nanosecond after the passage of the particle. As is stated in [59], “the main requirements for the trigger system are:

- discrimination on muon transverse momentum,
- bunch crossing identification,
- fast and coarse tracking information to be used in the high-level trigger stages,
• second φ coordinate measurement to complement the MDT measurement”,
• robustness towards random neutron and photon hits.

A scheme of the muon trigger system is shown in Figure 3.14. In the barrel region the triggering is performed by RPCs, providing a good spatial and time resolution and high rate capability. Three concentric cylindrical layers of RPCs are installed: two layers between the middle and the outer MDT layers, and one outside the MDTs. The triggering of muons with \( p_T \) ranging from 6 GeV to 35 GeV is possible. RPCs
are gaseous parallel electrode-plate detectors with two resistive plates at a distance of 2 mm operating at the voltage of 9.8 kV. Metallic strips for the measurement of \( \eta (\phi) \) with a pitch distance of 25 mm (30 mm) are mounted on the outer surfaces of the plates. Each layer of RPC consists of two independent detector layers. The measurement of \( \eta \) and \( \phi \) in all layers gives a total number of six coordinates. The detection efficiency per layer is 97% and the time resolution is 1.5 ns.

In the endcap regions, TGC installed in four planes around the beam pipe are used for triggering. The inner TGC plane is located at \( |z| \approx 7 \text{ m} \), three other TGC planes are mounted at a distance \( |z| \approx 14 \text{ m} \). The multilayer structure of the TGC gives in total 7 planes that a muon has to traverse. TGCs are multiwire proportional chambers filled with a gas mixture of CO\(_2\) (55%) and n–C\(_5\)H\(_{12}\) (45%). They are able to measure both the distance \( R \) to the beam pipe and the azimuthal coordinate \( \phi \). The TGCs can identify muons with more than 99% efficiency within 25 ns after the bunch crossing.

### 3.3 Trigger System

The total number of readout channels in the ATLAS detector is of the order of \( 10^8 \), and the approximate amount of data produced in each collision event is about 25 MB. At the LHC design luminosity of \( 10^{34} \text{ cm}^{-2}\text{s}^{-1} \) the bunch crossing rate reaches 40 MHz, with 22 proton-proton collisions in each bunch on the average. The total amount of produced data is way too huge to be stored, therefore an “on-the-fly” preselection of the events has to be made.

The reduction of data amount in the order of \( 10^7 \) is achieved with a three-step trigger system, set to choose the events with potentially interesting physics, while filtering out the background events. This system consists of Level-1 (L1) trigger, Level-2 (L2) trigger and the Event Filter (EF). L2 trigger and the EF together are called High Level Trigger (HLT). The three levels of the trigger system are working subsequently. The trigger selection scheme is presented in Figure 3.15.

First, the L1 hardware-based trigger reduces the data rate from 40 MHz down to 75 kHz. Its operation principle is bases on analyzing the bunch crossings using the information from the calorimeter and MS. The MS trigger reacts on events containing at least one muon, the logic provides six independently programmable \( p_T \) thresholds. The basic principle of the algorithm is to require a coincidence of hits in the different trigger stations along the path of a muon from the interaction point through the detector. The information used in the L1 trigger decision is the multiplicity of muons for each of the \( p_T \) thresholds.

The calorimeter is triggering on events with high energy deposits \( E_T \) or with large missing transverse energy \( E_{T\text{miss}} \). The pre-processor digitizes the analogue input signals, then uses a digital filter to associate them with specific bunch crossings. It uses a look-up table to produce the \( E_T \) values used for the trigger algorithms. The overall L1 accept decision is made by the central trigger processor, which combines the
information for different object types. The decision must reach the front-end electronics within 2.5 \mu s after the bunch crossing. Additional information on the L1 trigger can be found in [68].

If the L1 trigger has identified potentially interesting objects inside a certain region of the detector (called Region of Interest (RoI)), the L2 trigger is fed with the information on coordinates, energy, and type of signatures from the RoI, which represents only about 1-2% of the full event data. The details of HLT operation are described in [69]. The HLT applies trigger decisions in a series of steps. In each step the existing information is improved by acquiring additional data from increasingly more detector regions. A list of physics signatures (trigger chains), implemented event reconstruction (feature extraction) and selection algorithms are used to build signature and sequence tables for all HLT steps. Feature extraction algorithms attempt to identify features, like a track or a calorimeter cluster. Then, a hypothesis algorithm decides whether this feature meets the criteria (such as a shower shape, track-cluster match...
or $E_T$ threshold) necessary to continue. Each signature is tested in this way. The L2 trigger reduces the event rate down to 3.5 kHz, with an average event processing time of approximately 40 ms.

EF is the last stage of the online selection chain. It uses filtering algorithms based on the offline reconstruction with the basic principles of the event selection same as in L2. For those events which fulfill the selection criteria, a tag is added to the event data structure identifying into which physics stream the event has been classified. The average event processing time is 4 s and the event rate is reduced to 200 Hz. Events that have passed the EF are recorded for further offline analysis.

### 3.3.1 B-physics Triggers

ATLAS B-physics triggers have about 10% of the storage bandwidth devoted to all triggers. The triggering of $B$-meson decays without any muons in the final state is performed with the help of calorimeters, since there is no direct way to identify hadrons in ATLAS, and is only possible at the low luminosities [70]. The $B_0^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay events are identified with the muon trigger system (Section 3.2.5.2). Muonic trigger types relevant for this analysis are single-muon, two-muons, di-muon vertex and $B \rightarrow \mu^+ \mu^- X$, that are described in details in [71] and [70]. Depending on the muon $p_T$ threshold, the single- and double-muon triggers may need to be prescaled - that means, not every event will be recorded, since the rate can be too high. The $B \rightarrow \mu^+ \mu^- X$ trigger reconstructs both di-muon vertex and the vertex of the decaying $B$-meson.

In Figure 3.16 the performance of various muon triggers is presented, based on the data collected in the year 2011. There, the different trigger samples overlap, i.e. if an event fires several triggers it appears in several samples.

### 3.4 Data Processing and Storage

The average amount of data produced by the four LHC experiments is around 25 petabytes per year. To provide a storage and computing power for reconstruction and data analysis, the Worldwide LHC Computing Grid (WLCG) [73] was established.

The WLCG project is a global collaboration of more than 150 computing centers in nearly 40 countries. The centers are structured in the four stages of so called “Tiers”. Currently there is one Tier-0 directly at CERN, with 65000 processing cores and 30 PB of data storage on the disk, and another Tier-0 in Budapest, Hungary with 20000 cores and 5.5 PB of storage. In addition, there are 11 large Tier-1 centers and a lot of smaller Tier-2 and Tier-3. Tier-2′s are typically universities and other scientific institutes that can store sufficient data and provide adequate computing power for specific analysis tasks. Tier-3 computing resources consist of local clusters in a university department or even individual computers.
Figure 3.16: Invariant mass of oppositely charged muon candidate pairs selected by a variety of triggers [72].

The data selected by the EF (usually called Raw data (RAW)) is transferred to the Tier-0 and is archived on the CERN Advanced STORage Manager (CASTOR) system at CERN, and the first step of data processing also happens at the Tier-0, with Event Summary Data (ESD) as an output, stored on the CASTOR system. Tier-0 distributes the RAW data and the reconstructed output to the Tier-1 centers. Tier-1’s are responsible for the storage of a proportional share of RAW and reconstructed data, and for the further processing of the ESD to the so called Analysis Object Data (AOD) and the derived event metadata. The AOD datasets are further shared between the Tier-2 centers, that handle a proportional share of the production and reconstruction of simulated events.

More details of the data processing can be found in Chapter 4.
Chapter 4

Simulations and Software

In the following chapter, the general idea of MC simulations is outlined. Simulations of various processes, such as collision events (Section 4.4) and the interaction of particles with the detector material (Section 4.5.1, 4.5.2, 4.5.3) are an important part of every high energy physics experiment. In Section 4.5.4 the combination of full and fast detector simulations is discussed. The framework used for processing of real and simulated data is described in Section 4.3, and the software used for further physics analysis - in Section 4.8.1.

4.1 Monte-Carlo Simulations

In the most general meaning, Monte-Carlo (MC) methods are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. They are widely used in numerical analysis and simulations of natural phenomena.

MC simulations are a crucial part of modern particle physics. Here they serve to model certain physical processes, to calculate the propagation of the particles in various media, to predict the possible outcome of the experiment under the assumption of a certain model and so on. The development and testing of the analysis procedure is often done first on the simulated data, and only applied to the real data afterwards. The comparison of the real data with the simulated events is necessary to understand the detector, material effects, the background and interesting physics events. The simulated datasets must provide both sufficient statistics and a good agreement with the real data, so their production is a very CPU- and time-consuming task.
4.2 Simulation Chain

Several steps are required to obtain the simulated data that can be compared with the real recorded one. As it is necessary to have a common format for both of them, after a certain point the real and the simulated data have to go through the same stages of processing. The full simulation chain is presented in Figure 4.1, and the various steps will be discussed in more details in the following sections.

A lot of computing work in ATLAS, including MC simulations, data processing and physics analysis, is carried out in the Athena framework [74, 75].

4.3 Athena Framework

Athena is based on the Gaudi framework which was initially developed by the LHCb collaboration. It is used for steering of the various tasks related to simulation and data processing. The main design principles of Athena include [74]

- usage of abstract interfaces and dynamic libraries,
- a clear separation between data and algorithms,
- a clear separation between permanent and temporary data,
- the independence of the algorithmic code operating on the data on the technology used to store it, which might vary over the lifetime of the experiment, or depend on the local environment,
- usage of common components and interfaces.

Setting an Athena task (usually called a "job") starts with choosing the relevant settings in Python scripts called jobOptions. Athena jobs are configured by specifying a set of algorithms, services, and other com-
ponents to be used, as well as their properties. The C++ classes written by user to describe the desired
effect or a property can be easily integrated into a skeleton of an application within the framework. Var-
ious tasks related to event generation, detector simulation and reconstruction, and physics analyses can
be performed in Athena.

4.4 Event Generation

The first task in the queue of data simulation in particle physics is the MC event generation. Several
event generators exist on the market [76], with Pythia being the one most widely used by the LHC
collaborations.

At this step, by calculating various processes occurring in hadron-hadron collisions (see Section
2.5) the data samples are produced that resemble the outcome of the real collisions. For an adequate
description of a collision event, the full problem is factorized into a number of several subprocesses that
have to be simulated. They include the hard process, initial and final state radiations, multiple parton
interactions, beam remnants and hadronization. Due to the complexity of these processes, some event
generators use certain simplifications and must be tuned via a set of parameters.

The output of the event generator is stored in EVNT format.

4.4.1 Pythia Generator

A detailed information on the implementation of the event generation in Pythia can be found in [77]
and [78]. In this section, only the major points will be mentioned.

The current version, Pythia8, is fully written in C++, while the older Pythia6 used Fortran. The
software contains a library of hard processes and models for initial- and final-state parton showers,
multiple parton-parton interactions, beam remnants, string fragmentation and particle decays. It also
has a set of utilities and interfaces to external programs.

The protons are described with a set of PDFs according to their partonic content. The simulation of
a pp-collision in Pythia begins with two protons approaching each other. They start radiating partons
which subsequently split into even more partons. This process is called the initial-state shower. Inter-
action of two partons from different showers is called the hard process and usually leads to a creation
of two new partons (2 → 2 process). The created partons can branch into even more quarks and gluons,
creating a final-state shower. In the hadronization process, the partons get confined and form hadrons.
Baryons and mesons coming from the hadronization process may, in turn, decay into other particles.

The bottom quarks are created only in 1% of the events generated by Pythia. This makes the produc-
tion of MC samples for B-meson decays very time consuming and inefficient. Attempts to increase the
$b$-quark production rate directly by changing the corresponding parameters resulted in incompatibility with the experimental data from Fermilab. An interface to Pythia called PythiaB [79] was developed by the ATLAS collaboration in order to solve the problem of low statistics for $B$-physics studies. The principle idea of PythiaB is to filter the events at parton level, choosing for the further simulation only those that contain a $b$-quark in the phase space region defined by user. These events are then hadronized $k$ times. This of course can lead to a bias in the produced MC samples. To avoid this danger, $k$ is chosen in a way, that only one out of $k$ cloned events will pass all further filters and cuts, so in the final sample the events are not repeated.

PythiaB allows to speed up the $B$-physics simulations, to simulate only the channel(s) of interest, to apply the selection cuts at several stages of the simulation, and to define $b$-production parameters.

4.5 Detector Simulation

After the event generation, the produced particles must go through the next step, where their interactions with the detector material, as well as the response of the electronic components of the detector, are simulated. The detector simulation computes and stores the particle hits on sensitive detector elements. The hits information is stored in a data format called HITS.

The more precise the detector and electronics description is in the simulator, the better agreement with the real data can be achieved. However, this precision comes together with the increase of simulation time and CPU consumption. Several softwares used for the detector simulation in ATLAS are discussed in the next sections. They are: GEometry ANd Tracking (GEANT4), AtlFastII, Fast ATLAS Track Simulation (FATRAS) and Integrated Simulation Framework (ISF). An overview of various detector simulation strategies is given in [80].

4.5.1 GEANT4

GEANT4 is a C++ based software that is developed and maintained by the Geant4 collaboration [81]. It is used by many experiments also outside of high energy physics. It provides tools for handling geometry, tracking, detector response, run management and visualization. Currently, GEANT4 release 9.6 is used in ATLAS for the MC events production.

The simulation of various particle-matter interactions in GEANT4 is available for many particle types and different materials over a large range of particle energies. GEANT4 is also able to simulate the decays of unstable particles. The user may completely define the geometry of the material to be simulated and set a large number of parameters to obtain the desired behavior of the simulator. A detailed description of various physical processes and quantities in GEANT4 can be found in [82].
4.5. **DETECTOR SIMULATION**

The geometrical description of the ATLAS detector is decoupled from the simulation framework. A dedicated set of classes (GeoModel) provides the volume and material description, which is automatically translated into a GEANT4 description at the run time [83]. In Figure 4.2, the muon chambers are shown, as seen by GEANT4 after the conversion from GeoModel. The GeoModel also takes into account time-dependent misalignments of the geometrical description. This is achieved by storing the results of hardware alignment procedures in a Conditions Database and applying the misalignment values depending on the run number set by the user [83].

GEANT4 software is the most accurate and the most often used for the production of MC samples in the ATLAS experiment.

### 4.5.2 FATRAS

FATRAS is a software used for the fast simulation of processes inside ATLAS Inner Detector and Muon Spectrometer [84]. It uses a simplified detector description, a so called reconstruction geometry, where all detector components are described as thin discrete layers, contrary to the GEANT4 implementation. On Figure 4.3 photon conversion points in GEANT4 and FATRAS are shown, and the structure of detector material used in the simulators can be clearly seen. The material properties in FATRAS do not necessarily correspond to those of any real material, rather they are averaged quantities that allow to reproduce the real material effects sufficiently well. FATRAS also uses simplified parametrizations of physics processes.

Compared to GEANT4, FATRAS simulations are about 100 times faster. This speed-up is achieved
by spending less time on the simulation of secondary particles created in interactions with the detector material.

4.5.3 AtlFastII

AtlFastII combines a fast calorimeter simulation implemented in FastCaloSim [85] with the so called full (GEANT4) simulation of the ID and MS. In AtlFastII, muons are the only particles that are simulated by GEANT4 and can reach the MS. FastCaloSim module does not transport the particles through the calorimeter, therefore the punch-through effect (see Section 4.5.4.1) is not accounted for in AtlFastII. The energy of single particle showers in the calorimeter is deposited directly using parametrizations of their longitudinal and lateral energy profile, which can be tuned against data to gain more accuracy. The output information includes the energies in the calorimeter cells.

A factor of 10 speed-up is gained compared to the full GEANT4 simulation. The AtlFastIIIF simulator, which combines the fast simulation of the ID, calorimeter and MS is about two orders of magnitude faster than the GEANT4.
4.5. DETECTOR SIMULATION

Simulations can be very demanding on the computing power, so a compromise between the precision and the simulation time has to be found. In many physics studies, only certain type of particles or events need to be simulated with high precision, while the rest can approximately resemble the real behavior. For instance, one can be interested in a precise simulation of muon tracks only inside the ID and MS, above certain $p_T$ threshold.

The basic idea of the ISF is to allow to run all the types of simulation in one job [80]. The user may fully configure the job properties, and choose the particle type(s), subdetector and the phase space region that will be simulated by GEANT4, while the rest will be handled by the fast simulators (FATRAS and AtlFastII). This allows to find an optimal balance between the precision and execution time, according to the specific requirements of every analysis. A use-case example of one simulated event is shown in Figure 4.4 to visualize the basic principle of the ISF.

The ISF can incorporate new simulation types in the future and can be adjusted to the usage of parallel computing techniques. Currently the framework is still under development, but it is expected to be ready for MC production in the year 2015.
4.5.4.1 Parametrized Simulation of the Punch-Through Effect

One of the features offered in ISF is the simulation of punch-through effect. It occurs when a high energy hadron shower is not fully stopped in the calorimeter, but continues to traverse further, creating "fake" signatures in the MS. These hits are fake in a sense that they were not created by muons. A similar type of event is a decay-in-flight, when a hit in the MS was produced by a muon, but the associated track in the ID belongs to a hadron (most often, a pion). Depending on the properties of the punch-through particle (particle type, energy, charge, position), the effect on the physics analysis may be significant.

The simulation of the punch-through effect inside the ISF is performed by the PunchThroughTool, which is a part of FastCaloSim package. Its development was a part of author’s work in the ATLAS collaboration, although not directly related to the topic of this thesis. It is based on the parametrized simulation in FATRAS [86] and was rewritten to use the ISF classes and the updated detector geometry.

The physics model of the particle interaction with the calorimeter is not simulated by the tool, only the results of the interaction processes are reproduced. A number of input parameters are given to the PunchThroughTool, including particle type, initial energy, position and direction of flight. The punch-through probability and the properties of the outcoming particles are functions of the input parameters, stored in a look-up table. The functions in the look-up table are obtained from the fit to the corresponding distributions produced in GEANT4 simulation. The outcoming particle appears at the MS entry at the same position, which the initial particle would reach, if it propagated through the calorimeter along a straight line.

The fast punch-through simulation outcome has to agree sufficiently well with the full GEANT4 simulation. For that a number of parameters can be adjusted.

4.6 Digitization

After the detector simulation, the stored particle hits need to be translated into a data format which corresponds to the format retrieved from the detector. This is done in a digitization process. Digitized simulated data correspond to bytestream converted data obtained from the detector and is stored in the RAW Data Object (RDO) format. All further steps of data processing need to be carried out the same way for real and simulated data.

During the digitization, the interaction of the particle with the material is transformed into measurable quantities, such as charge or energy collected by the detector modules, and this information is stored. For the simulated data, the MC truth information is available in addition. The so called pile-up simulation, which accounts for the fact of multiple proton-proton collisions in a single event, is also
4.7 Reconstruction

The RDO is further processed to connect the stored physical quantities with the real (or simulated) particles, that is, to translate the information stored in the RDO into particle track, type and energy. To find a particle track means to connect the detector hits with a realistic trajectory. Particle energy and momentum are obtained from the track curvature and calorimeter measurements. Several reconstruction algorithms exist, that use different approaches to perform this task. The missing transverse energy, which has no direct signal in the detector, is interpreted in the reconstruction as either neutrinos or supersymmetric particles.

One of the main tasks of the reconstruction step is the reduction of the stored data. The reconstructed data is stored in the Event Summary Data (ESD) format, that needs about three times less space than RDO. It can be reduced even more by omitting the objects that are uninteresting for a specific analysis, which results in the AOD data format. Although the user can work directly with the AOD, a further data reduction can be achieved by creating a Derived Physics Data (DPD), where only certain properties of the objects of interest are stored.

All of the reconstructed data formats are readable by ROOT software (see Section 4.8.1) and can be easily used for physics analysis.

4.8 Data Analysis Software

4.8.1 ROOT

ROOT is an object-orientated framework for data processing developed at CERN, designed to work efficiently with the large amounts of data [87]. The data is defined as a set of objects, and specialized storage methods are used to get direct access to the separate attributes of the selected objects, without having to include the rest of the data in the analysis. ROOT provides a convenient way to store and access the relevant event information and to visualize the results of the analysis. It contains methods to create histograms in an arbitrary number of dimensions, perform curve fitting, function evaluation, minimization, multivariate classification (Toolkit for Multivariate Analysis (TMVA) package), visualize and plot graphics objects. Particularly useful for the analysis described in this thesis is the RooFit package which allows the user to perform complex data modeling and fitting. ROOT is an open system that
can be dynamically extended by linking external libraries and therefore can be adjusted to better serve the needs of an individual analysis.
Chapter 5

Measurement

In this chapter, analysis of 4.9 fb⁻¹ of data collected with the ATLAS detector in the year 2011, is described. In Section 5.1 data and MC samples used for the analysis, are listed. In Section 5.2 the applied requirements are described. Section 5.4 represents the construction of the model and the fit procedure performed to obtain the values of angular observables of the $B^0_{d} \rightarrow K^{*0} \mu^+ \mu^-$ events from the data sample. Section 5.6 is dedicated to the study of systematic uncertainties. The final results are presented in Section 6.1.

5.1 Data and Monte-Carlo Samples

The total size of the data collected by ATLAS amounts to 4.9 fb⁻¹. About 172 million events that were recorded in the muon stream (see Section 3.3) were used in this analysis. The used MC samples include signal channel $B^0_{d} \rightarrow K^{*0} \mu^+ \mu^-$ and various background sources listed in Table 5.1. PythiaB (see Section 4.4.1) was used for the generation of most of the samples, except for Drell-Yan events ($q\bar{q}$-annihilation with subsequent production of $\ell\ell$ pair), where Pythia was used. The simulation of detector effects was performed by GEANT4 (Section 4.5.1).

The $b\bar{b} \rightarrow \mu^+ \mu^- X$ type of processes with two muons and several hadronic tracks in the final state constitute the largest part of the background events. Since there is no dedicated detector for the hadron identification, charged hadronic tracks in these events can be misidentified as kaon and pion and the event may look like signal. The invariant mass distribution of $B^0_{d}$-candidates in this type of events follows exponential shape and the angular distribution has no distinct structure.

The so called “peaking” background contributions are coming from $B^0_{d} \rightarrow K^{*0}J/\psi$ and $B^0_{d} \rightarrow K^{*0}\psi(2S)$ decays. The invariant mass distribution of $B^0_{d}$-candidates in these decays has a Gaussian shape with a mean value at $B^0_{d}$-meson mass, same as the signal decay $B^0_{d} \rightarrow K^{*0} \mu^+ \mu^-$. In the di-muon
mass spectrum, distinct peaks can be seen around the PDG masses of $J/\psi$ and $\psi(2s)$ mesons, respectively. These decays also have a certain shape of the angular distributions, that differs from flat [88].

Fake $B_{d}^{0}$-decays can also be reconstructed from $c \bar{c} \rightarrow \mu^{+} \mu^{-} X$ decays and Drell-Yan processes that were found to have roughly exponential and flat invariant mass distribution, respectively.

All MC samples were generated with a cut on the muons transverse momentum $p_{T}$ to account for the energy loss of the muons before reaching the MS. Common cuts are $p_{T}(\mu_{1,2}) > 2.5$ GeV, $p_{T}(\mu_{1,2}) > 3.5$ GeV and $p_{T}(\mu_{1,2}) > 4.0$ GeV. Samples available for this analysis are listed in Table 5.1 together with the total number of generated events $N_{ev}$, $p_{T}(\mu_{1,2})$ setting for both muons and the total cross section calculated by PythiaB (or Pythia for the Drell-Yan process).

In order to account for the differences in the $p_{T}(\mu_{1,2})$ settings in the various samples, the renormalization factors had to be calculated as described in Section 5.1.1.

5.1.1 Monte-Carlo Normalization

The sample with the highest statistics is $b \bar{b} \rightarrow \mu^{+} \mu^{-} X$ containing $198.6 \cdot 10^{6}$ events. The value of $p_{T}(\mu_{1,2})$ threshold of this sample should be used as the standard value. To avoid losing events when applying this $p_{T}$ cut to the $B_{d}^{0} \rightarrow K^{*0} \mu^{+} \mu^{-}$ and $c \bar{c} \rightarrow \mu^{+} \mu^{-} X$ samples, they have to be selected with $p_{T}(\mu_{1,2}) > 4.0$ rather than $p_{T}(\mu_{1,2}) > 2.5$ GeV. The 0.5 GeV difference in the $p_{T}(\mu_{1,2})$ threshold between these samples and the $b \bar{b} \rightarrow \mu^{+} \mu^{-} X$ sample is taken into account by upscaling the two samples. The upscaling factors $U_{B_{d}^{0} \rightarrow K^{*0} \mu^{+} \mu^{-}}$ and $U_{c \bar{c} \rightarrow \mu^{+} \mu^{-} X}$ are calculated by looking at the corresponding samples with $p_{T}(\mu_{1,2}) > 2.5$ GeV and calculating the number of events left after applying $p_{T}(\mu_{1,2}) > 3.5$ GeV and

<table>
<thead>
<tr>
<th>Process</th>
<th>$N_{ev}$, $10^{6}$</th>
<th>$p_{T}(\mu_{1,2})$ threshold</th>
<th>Generated cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \bar{b} \rightarrow \mu^{+} \mu^{-} X$</td>
<td>198.6</td>
<td>3.5 GeV</td>
<td>0.08 $\mu$b</td>
</tr>
<tr>
<td>$b \bar{b} \rightarrow \mu^{+} \mu^{-} X$</td>
<td>5.8</td>
<td>2.5 GeV</td>
<td>0.66 $\mu$b</td>
</tr>
<tr>
<td>$B_{d}^{0} \rightarrow K^{*0} J/\psi$</td>
<td>0.5</td>
<td>2.5 GeV</td>
<td>0.25 $\mu$b</td>
</tr>
<tr>
<td>$c \bar{c} \rightarrow \mu^{+} \mu^{-} X$</td>
<td>4.9</td>
<td>4.0 GeV</td>
<td>0.02 $\mu$b</td>
</tr>
<tr>
<td>$c \bar{c} \rightarrow \mu^{+} \mu^{-} X$</td>
<td>2.9</td>
<td>2.5 GeV</td>
<td>0.25 $\mu$b</td>
</tr>
<tr>
<td>Drell-Yan</td>
<td>0.5</td>
<td>2.5 GeV</td>
<td>0.17 $\mu$b</td>
</tr>
<tr>
<td>$B_{d}^{0} \rightarrow K^{*0} \mu^{+} \mu^{-}$</td>
<td>0.2</td>
<td>4.0 GeV</td>
<td>7.14 $\mu$b</td>
</tr>
<tr>
<td>$B_{d}^{0} \rightarrow K^{*0} \mu^{+} \mu^{-}$</td>
<td>0.2</td>
<td>2.5 GeV</td>
<td>35.01 $\mu$b</td>
</tr>
</tbody>
</table>

Table 5.1: Available MC samples.
5.1. DATA AND Monte-Carlo SAMPLES

The normalization procedure in case of the \( K \rightarrow d \rightarrow \mu^+ \mu^- \) sample can be calculated simply as

\[
\mathcal{U}_{K \rightarrow d \rightarrow \mu^+ \mu^-} = \frac{N_{K \rightarrow d \rightarrow \mu^+ \mu^-}(p_T(\mu_{1,2}) > 4.0 \text{ GeV})}{N_{K \rightarrow d \rightarrow \mu^+ \mu^-}(p_T(\mu_{1,2}) > 3.5 \text{ GeV})}
\]

(5.1)

\[
\mathcal{U}_{c \bar{c} \rightarrow \mu^+ \mu^- \rightarrow X} = \frac{N_{c \bar{c} \rightarrow \mu^+ \mu^-}(p_T(\mu_{1,2}) > 4.0 \text{ GeV})}{N_{c \bar{c} \rightarrow \mu^+ \mu^-}(p_T(\mu_{1,2}) > 3.5 \text{ GeV})}
\]

(5.2)

The \( B_3^0 \rightarrow K^0 J/\psi \), \( c \bar{c} \rightarrow \mu^+ \mu^- \), Drell-Yan and \( B_3^0 \rightarrow K^{*0} \mu^+ \mu^- \) samples have first to be normalized to the same integrated luminosity as the real recorded data, i.e. 4.9 fb\(^{-1}\). Since the \( b \bar{b} \rightarrow \mu^+ \mu^- \) sample is upscaled to the same integrated luminosity as the real recorded data, the normalization of the \( B_3^0 \rightarrow K^{*0} J/\psi \) sample can be calculated simply as

\[
\mathcal{C}_{B_3^0 \rightarrow K^{*0} J/\psi} = \frac{N_{B_3^0 \rightarrow K^{*0} J/\psi}}{N_{B_3^0 \rightarrow K^{*0} J/\psi}}
\]

(5.3)

where \( N_{B_3^0 \rightarrow K^{*0} J/\psi} \) is the number of \( B_3^0 \rightarrow K^{*0} J/\psi \) events in the \( b \bar{b} \rightarrow \mu^+ \mu^- \) sample.

For \( c \bar{c} \rightarrow \mu^+ \mu^- \) and Drell-Yan samples, the normalization factors are calculated by using the corresponding cross sections and event numbers

\[
\mathcal{C}_{c \bar{c} \rightarrow \mu^+ \mu^-} = \frac{\sigma_{c \bar{c} \rightarrow \mu^+ \mu^-}}{\sigma_{b \bar{b} \rightarrow \mu^+ \mu^-}} \cdot \frac{N_{B_0^0 \rightarrow \mu^+ \mu^-}}{N_{c \bar{c} \rightarrow \mu^+ \mu^-}}
\]

(5.4)

\[
\mathcal{C}_{DY} = \frac{\sigma_{DY}}{\sigma_{b \bar{b} \rightarrow \mu^+ \mu^-}} \cdot \frac{N_{B_0^0 \rightarrow \mu^+ \mu^-}}{N_{DY}}
\]

(5.5)

The normalization procedure in case of \( B_0^0 \rightarrow K^{*0} J/\psi \) decays by the branching fractions of \( B_0^0 \rightarrow K^{*0} \mu^+ \mu^- \), \( B_0^0 \rightarrow K^{0} J/\psi \) and \( J/\psi \rightarrow \mu^+ \mu^- \) processes taken from the PDG [16]:

\[
\mathcal{B}(B_0^0 \rightarrow K^{*0} \mu^+ \mu^-) = (1.06 \pm 0.1) \cdot 10^{-6}
\]

(5.6)

\[
\mathcal{B}(B_0^0 \rightarrow K^0 J/\psi) = (1.34 \pm 0.06) \cdot 10^{-3}
\]

(5.7)

\[
\mathcal{B}(J/\psi \rightarrow \mu^+ \mu^-) = (5.93 \pm 0.06) \cdot 10^{-2}
\]

(5.8)

Therefore, the normalization factor for the signal sample is

\[
\mathcal{C}_{B_0^0 \rightarrow K^{*0} \mu^+ \mu^-} = \frac{N_{B_0^0 \rightarrow K^{*0} \mu^+ \mu^-}}{N_{B_0^0 \rightarrow K^{*0} \mu^+ \mu^-}} \cdot \frac{\mathcal{B}(B_0^0 \rightarrow K^{*0} \mu^+ \mu^-)}{\mathcal{B}(B_0^0 \rightarrow K^0 J/\psi) \cdot \mathcal{B}(J/\psi \rightarrow \mu^+ \mu^-)}
\]

(5.9)

To upscale all samples to 4.9 fb\(^{-1}\), the normalization factors are multiplied by

\[
\frac{\sigma_{b \bar{b} \rightarrow \mu^+ \mu^-}}{N_{b \bar{b} \rightarrow \mu^+ \mu^-}} \cdot 4.9 \text{ fb}^{-1}
\]

(5.10)
and the final normalization factors become
\[ C_{b \bar{b} \rightarrow \mu^+ \mu^- - X} = 4.9 \text{ fb}^{-1}, \quad \frac{\sigma_{b \bar{b} \rightarrow \mu^+ \mu^- - X}}{N_{b \bar{b} \rightarrow \mu^+ \mu^- - X}} \]
\[ C_{B^0 \rightarrow K^{*0} / \psi} = 4.9 \text{ fb}^{-1}, \quad \frac{\sigma_{b \bar{b} \rightarrow \mu^+ \mu^- - X}}{N_{b \bar{b} \rightarrow \mu^+ \mu^- - X}} \cdot \tilde{C}_{B^0 \rightarrow K^{*0} / \psi} \]
\[ C_{c \bar{c} \rightarrow \mu^+ \mu^- - X} = 4.9 \text{ fb}^{-1}, \quad \frac{\sigma_{b \bar{b} \rightarrow \mu^+ \mu^- - X}}{N_{b \bar{b} \rightarrow \mu^+ \mu^- - X}} \cdot \tilde{C}_{c \bar{c} \rightarrow \mu^+ \mu^- - X} \cdot U_{c \bar{c} \rightarrow \mu^+ \mu^- - X} \]
\[ C_{DY} = 4.9 \text{ fb}^{-1}, \quad \frac{\sigma_{b \bar{b} \rightarrow \mu^+ \mu^- - X}}{N_{b \bar{b} \rightarrow \mu^+ \mu^- - X}} \cdot \tilde{C}_{DY} \]
\[ C_{B^0 \rightarrow K^{*0} \mu^+ \mu^-} = 4.9 \text{ fb}^{-1}, \quad \frac{\sigma_{b \bar{b} \rightarrow \mu^+ \mu^- - X}}{N_{b \bar{b} \rightarrow \mu^+ \mu^- - X}} \cdot \tilde{C}_{B^0 \rightarrow K^{*0} \mu^+ \mu^-} \cdot U_{B^0 \rightarrow K^{*0} \mu^+ \mu^-} \]

The corresponding numerical values are listed in Table 5.2.

<table>
<thead>
<tr>
<th>Normalization factor</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{b \bar{b} \rightarrow \mu^+ \mu^- - X}$</td>
<td>$1.94 \pm 0.06$</td>
</tr>
<tr>
<td>$C_{B^0 \rightarrow K^{*0} / \psi}$</td>
<td>$1.94 \pm 0.06$</td>
</tr>
<tr>
<td>$C_{c \bar{c} \rightarrow \mu^+ \mu^- - X}$</td>
<td>$38.46 \pm 1.16$</td>
</tr>
<tr>
<td>$C_{DY}$</td>
<td>$(1.70 \pm 0.05) \cdot 10^3$</td>
</tr>
<tr>
<td>$C_{B^0 \rightarrow K^{*0} \mu^+ \mu^-}$</td>
<td>$(9.61 \pm 1.07) \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

Table 5.2: Normalization factors of the MC samples.

5.2 Event Selection and Reconstruction

The selection of events in the real data is based on their topology: signal-like events should have two muon tracks and two hadronic tracks, coming from the same decay vertex. In reality the hadronic tracks are coming from $K^{*0}$ decay vertex, but due to the very short lifetime of $K^{*0}$, $\tau \approx 10^{-20}$ s, the detector cannot resolve the two vertices. Additional requirements are applied to ensure high quality of the measurements and to maximize the number of signal events while getting rid of most of the background ones, as will be discussed in the next sections.

5.2.1 Rejection of the Charmonium Resonances

The decays with the same final state ($B^0_d \rightarrow K^{*0} (\rightarrow K^+ \pi^-) J/\psi (\rightarrow \mu^+ \mu^-)$ and $B^0_d \rightarrow K^{*0} (\rightarrow K^+ \pi^-) \psi(2s) (\rightarrow \mu^+ \mu^-)$) cannot be distinguished from the signal channel $B^0_d \rightarrow K^{*0} (\rightarrow K^+ \pi^-) \mu^+ \mu^-$ by the topology selection. Moreover, the $B^0_d \rightarrow K^{*0} J/\psi$ channel has a roughly 1000 times higher branching fraction compared to $B^0_d \rightarrow K^{*0} \mu^+ \mu^-$, so the rejection of these events is a crucial task in the analysis.
5.2. EVENT SELECTION AND RECONSTRUCTION

The invariant mass of the two muons originating from a $J/\psi$ or $\psi(2S)$ decays lies around the PDG mass of the corresponding charmonium resonance, with the width defined by the detector resolution. The events with di-muon mass inside the respective regions are rejected from the analysis. Since the detector resolution depends on the pseudorapidity of the muon tracks, the whole data sample was split into 3 categories:

- barrel-barrel (BB), with $|\eta| < 1.05$ for both muon tracks
- endcap-endcap (EE), with $1.05 < |\eta| < 2.5$ for both muon tracks
- barrel-endcap (BE), if one muon track belongs to the barrel region $|\eta| < 1.05$, while the other is in the endcap $1.05 < |\eta| < 2.5$.

In each region, a maximum likelihood fit of the di-muon mass spectra was performed, with the charmonium resonances described by a Gaussian function, and the background component described by a linear function:

$$L = \prod_{i=1}^{N} \left[ f \cdot (f_{J/\psi} \cdot M_{J/\psi}(m_i) + (1 - f_{J/\psi}) \cdot M_{\psi(2S)}(m_i)) + (1 - f) M_{\text{comb}}(m_i) \right]$$  \hspace{1cm} (5.12)

where $f$ is the fraction of $J/\psi$ and $\psi(2S)$ events together and $f_{J/\psi}$ is the fraction of $J/\psi$ events alone,

$$M_{J/\psi} = \frac{1}{\sqrt{2\pi}\sigma_{J/\psi}} \exp \left(-\frac{(m_i - m_{J/\psi})^2}{2\sigma_{J/\psi}^2}\right),$$  \hspace{1cm} (5.13)

$$M_{\psi(2S)} = \frac{1}{\sqrt{2\pi}\sigma_{\psi(2S)}} \exp \left(-\frac{(m_i - m_{\psi(2S)})^2}{2\sigma_{\psi(2S)}^2}\right),$$

$$M_{\text{bckg}}(m_i) = p \cdot m_i$$

and $m_i$ is the di-muon invariant mass. The mean and width of the Gaussians as well as the linear coefficient $p$ are obtained from the fits to the real data and are listed in the Table 5.3 for each resolution category. The fitted di-muon mass spectra are shown in Figure 5.1, Figure 5.2 and Figure 5.3.

All events within $3\sigma$ range from the mean value in each resolution category are excluded from further analysis. The loss of signal events due to this exclusion is estimated to be about 30.31%. The study of $B_d^0 \rightarrow K^0 J/\psi$ MC sample also shows that 2.27% of these events survive the exclusion at $3\sigma$ level. Even this relatively small number is large enough to pollute the signal channel. Further reduction of this type of background in the “tails” of the charmonium peaks is discussed in Section 5.2.3.

5.2.2 Baseline Cuts

Cuts are the requirements, or limits, applied to the values of certain variables. The aim of the baseline is to insure high quality of the measured properties by rejecting e.g., random combinations of hits in the
### Table 5.3: Results of di-muon spectra fits. Errors are statistical only.

<table>
<thead>
<tr>
<th>Fit parameter</th>
<th>Fitted value (BB-muons)</th>
<th>Fitted value (BE-muons)</th>
<th>Fitted value (EE-muons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{J/\psi}$</td>
<td>3094.0 ± 0.2 MeV</td>
<td>3095.4 ± 0.26 MeV</td>
<td>3096.9 ± 0.4 MeV</td>
</tr>
<tr>
<td>$\sigma_{J/\psi}$</td>
<td>38.67 ± 0.17 MeV</td>
<td>52.84 ± 0.22 MeV</td>
<td>75.22 ± 0.37 MeV</td>
</tr>
<tr>
<td>$m_{\psi(2S)}$</td>
<td>3678.7 ± 1.7 MeV</td>
<td>3682.3 ± 2.0 MeV</td>
<td>3682.5 ± 3.7 MeV</td>
</tr>
<tr>
<td>$\sigma_{\psi(2S)}$</td>
<td>43.63 ± 1.72 MeV</td>
<td>52.84 ± 0.22 MeV</td>
<td>77.94 ± 3.87 MeV</td>
</tr>
<tr>
<td>$p$</td>
<td>$-(0.49 \pm 0.01)$</td>
<td>$-(0.24 \pm 0.01)$</td>
<td>$-(0.57 \pm 0.01)$</td>
</tr>
<tr>
<td>$f$</td>
<td>0.690 ± 0.002</td>
<td>0.691 ± 0.002</td>
<td>0.689 ± 0.003</td>
</tr>
<tr>
<td>$f_{J/\psi}$</td>
<td>0.959 ± 0.001</td>
<td>0.941 ± 0.002</td>
<td>0.960 ± 0.002</td>
</tr>
</tbody>
</table>

Figure 5.1: Fit of di-muon mass spectrum in the barrel-barrel region.
5.2. EVENT SELECTION AND RECONSTRUCTION

Figure 5.2: Fit of di-muon mass spectrum in the barrel-endcap region.

Figure 5.3: Fit of di-muon mass spectrum in the endcap-endcap region.
detector modules that could be by chance reconstructed as a track, or random combinations of tracks looking like if they were coming from the same vertex. The same baseline cuts are usually used in similar analyses.

Baseline cuts used here are taken from the $B^0_d \to K^{*0}/\psi$ analysis \cite{89} since the final state of $B^0_d \to K^{*0}/\psi$ decay is same as of $B^0 \to K^{*0}\mu^+\mu^-$. These requirements are the following:

- all four tracks in the final state are required to have $|\eta| < 2.5$ to account for the detector acceptance effects,
- muon pairs refitted to a common vertex must satisfy $\chi^2_{\text{n.d.f}} < 10$ to exclude random combinations of the tracks,
- only events with invariant mass of $(K\pi)$-system satisfying $846 \text{ MeV} < m_{\text{inv}}(K\pi) < 946 \text{ MeV}$ are taken ($m(K^{*0}) = 895.94 \text{ MeV}$ \cite{16}),
- $p_T$ of kaons and pions must be above 0.5 GeV and the $p_T$ of muons above 3.5 GeV.

The variable $\chi^2_{\text{n.d.f}}$ refers to the $\chi^2$ distribution of the reconstructed $B^0_d$ vertex position.

The efficiency of the baseline cuts on MC samples is shown in Table 5.4. It is defined as the number of events in the sample after the cuts divided by the number of events in the same sample with a $p_T(\mu_{1,2}) > 3.5$ GeV cut, to account for the different MC generation settings in the efficiency calculation.

<table>
<thead>
<tr>
<th>Cut</th>
<th>$\epsilon_{\text{sig}}$</th>
<th>$\epsilon_{B^0_d \to K^{*0}/\psi}$</th>
<th>$\epsilon_{B^0 \to K^{*0}\mu^+\mu^-}$</th>
<th>$\epsilon_{\text{DY}}$</th>
<th>$\epsilon_{B^0 \to K^{*0}/\psi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>tracks $</td>
<td>\eta</td>
<td>&lt; 2.5$</td>
<td>99.2%</td>
<td>98.3%</td>
<td>99.1%</td>
</tr>
<tr>
<td>$\chi^2_{\text{n.d.f}}(\mu^+\mu^-) &lt; 10.$</td>
<td>99.2%</td>
<td>91.9%</td>
<td>89.7%</td>
<td>99.0%</td>
<td>93.7%</td>
</tr>
<tr>
<td>$846 \text{ MeV} &lt; m_{\text{inv}}(K\pi) &lt; 946 \text{ MeV}$</td>
<td>56.0%</td>
<td>49.7%</td>
<td>49.8%</td>
<td>50.0%</td>
<td>54.2%</td>
</tr>
<tr>
<td>$p_T(K) &gt; 0.5$ GeV</td>
<td>99.4%</td>
<td>99.0%</td>
<td>99.3%</td>
<td>98.1%</td>
<td>99.1%</td>
</tr>
<tr>
<td>$p_T(\pi) &gt; 0.5$ GeV</td>
<td>99.4%</td>
<td>99.1%</td>
<td>99.3%</td>
<td>98.2%</td>
<td>99.1%</td>
</tr>
<tr>
<td>All cuts</td>
<td>54.9%</td>
<td>44.4%</td>
<td>43.9%</td>
<td>46.7%</td>
<td>49.8%</td>
</tr>
</tbody>
</table>

Table 5.4: Efficiency of the baseline cuts on the MC samples.

### 5.2.3 Selection Cuts

Selection cuts are the requirements specific to every analysis. Here the variables worth looking at must exhibit a different behavior for the signal and background channels. In some cases, a certain cut can be useful to reject only a particular type of background, leaving other background events largely unaffected. After a suitable set of discriminating variables is found, the cut values should be optimized to
achieve best signal significance. The optimization procedure is described in Section 5.2.4. The selection variables are listed below.

- A cut on $B^0_d$ meson lifetime ($\tau_{B^0_d} = (1.519 \pm 0.007) \cdot 10^{-12}$ s [16]) can remove events with short lived hadrons. Such events are mainly included in $b\bar{b} \rightarrow \mu^+\mu^-X$, $c\bar{c} \rightarrow \mu^+\mu^-X$ and Drell-Yan samples. The usage of lifetime significance $\tau/\Delta \tau$ allows in addition to choose $B^0_d$-candidates with the smallest uncertainty of the lifetime measurement.
- Further reduction of hadronic background is achieved by a cut on the 3D pointing angle $\theta$. As shown in Figure 5.4, $\theta$ is the angle between the reconstructed direction of flight of the $B^0_d$-candidate and its 3-momentum vector. For the real $B^0_d$-decays this angle should be equal zero, within the detector resolution, consequently, $\cos \theta$ should be very close to one.
- A certain lower limit on $K^{*0}$-candidates transverse momentum $p_T$ insures that the particle is created in a $B^0_d$-decay.
- A requirement on $B^0_d$-vertex reconstruction and fitting is applied, so that $\chi^2_{n.d.f}(B^0_d)$ is below a certain value.
- The last cut on $|\langle M(B^0_d)_{\text{rec}} - M(B^0_d)_{\text{PDG}} \rangle - \langle M(\mu\mu)_{\text{rec}} - M(J/\psi)_{\text{PDG}} \rangle| > \Delta M$ was used in $B^0_d \rightarrow K^{*0}\mu^+\mu^-$ analysis performed by the CDF collaboration [90] together with the constraint (\langle M(\mu\mu)_{\text{rec}} < M(J/\psi)_{\text{PDG}} \rangle) to remove radiative charmonium decays (e.g. $J/\psi \rightarrow \mu^+\mu^-\gamma$) that originate from the corresponding $B^0_d$-decays (e.g. $B^0_d \rightarrow K^{*0}J/\psi$). When the four tracks are fitted to a common vertex and the photon escapes undetected, the invariant mass of reconstructed $B^0_d$-candidate will be lower than the PDG value, and a peaking structure will be observed in the invariant mass spectrum due to such partially reconstructed decays. In this analysis it was found that applying this cut without the (\langle M(\mu\mu)_{\text{rec}} < M(J/\psi)_{\text{PDG}} \rangle) constraint removes some of the remaining $B^0_d \rightarrow K^{*0}J/\psi$ and $B^0_d \rightarrow K^{*0}\psi(2S)$ events. In the following, this cut will be referred to as $\Delta M$.  

![Figure 5.4: Definition of the pointing angle $\theta$. PV - primary vertex (collision point), SV - secondary (decay) vertex.](image)
In Figure 5.5 the distribution of events is shown as a function of the $B^0_d$ mass and the di-muon mass after all the baseline and selection cuts have been applied. Here one can see the regions excluded due to the $J/\psi$ and $\psi(2S)$ resonances as well as the lack of events in two regions diagonally raising from low $B^0_d$ mass. These events have been removed with the $\Delta M$ cut. Therefore, in every $q^2$ bin the mass fit range had to be chosen individually, so that the background shape is the least affected by this cut. This issue is discussed in Section 5.6.3 and the chosen fit ranges are presented in Table 5.13.

A few more sources of background events were considered when the set of selection variables was chosen. They include

- Background from the decay channel $B^0_s \rightarrow J/\psi \phi$, where the decay $\phi \rightarrow K^+ K^-$ has been misidentified as $K^{*0} \rightarrow K^+ \pi^-$ (i.e. a kaon track is misidentified as a pion track). If the invariant mass of these tracks would fall into a region $1.0085 \text{ GeV} < m(K,K) < 1.0305 \text{ GeV}$ the event would be rejected. No such event was found in the final data sample.
- Background due to $B^\pm \rightarrow K^\pm \mu^+ \mu^-$ events. An additional requirement was tested: the di-muon pairs were combined with each charged hadronic track under the hypothesis that the three tracks originate from $B^\pm$ decay. As shown in Figure 5.6, the $B^\pm$ candidates mass spectrum shows background-like behavior without an obvious peak at the $B^\pm$ mass ($m_{B^\pm} = 5279.25 \text{ MeV}$ [16]), so it is unlikely that the three tracks are indeed coming from a $B^\pm$ decay. Therefore it was decided not to apply this
5.2. EVENT SELECTION AND RECONSTRUCTION

5.2.4 Optimization of Selection Cuts

To optimize the values of the selection cuts an estimator has to be chosen to quantify their efficiency. Optimal requirements should reduce the amount of background events while retaining most of the signal events. In this analysis, the estimator \( P(S, B) \) is chosen as

\[
P(S, B) = \frac{S}{\sqrt{S + B}},
\]

where \( S \) and \( B \) denote the number of signal and background events left after a particular combination of cuts. With sufficiently large data sample, the approximation for the total number of events \( N \to \infty \) can be used and the 5.14 can then be rewritten as

\[
P(N, B) = \frac{N - B}{\sigma},
\]

where \( \sigma \) is the standard deviation.

To find the optimal values of the selection parameters, each of them was limited to a certain reasonable range, and a discrete scan through such parameter space was performed. The space of cut parameters is listed in Table 5.5.

The selection optimization was performed using the MC samples introduced in Section 5.1. The \( B^0_d \) mass region was chosen between 5050 MeV and 5510 MeV, because in larger sidebands, efficient...
suppression of the high combinatorial background would require very tight \( \tau/\Delta\tau \) cuts and would therefore strongly reduce the number of surviving signal events. The final number of events in each sample surviving after each cut combination consists of raw events from the samples weighted by the corresponding scaling coefficients (see Table 5.2):

\[
S = C_{B_0^d \to K^*\mu^+\mu^-} \cdot N_{B_0^d \to K^*\mu^+\mu^-}, \\
B = C_{b\bar{b} \to \mu^+\mu^- X} \cdot N_{b\bar{b} \to \mu^+\mu^- X} + C_{B_0^d \to K^*\psi} \cdot N_{B_0^d \to K^*\psi} + \\
+ C_{c\bar{c} \to \mu^+\mu^- X} \cdot N_{c\bar{c} \to \mu^+\mu^- X} + C_{DY} \cdot N_{DY}. \tag{5.16}
\]

The highest estimator value corresponds to the following cuts:

- \( \tau/\Delta\tau (B_0^d) > 12.75 \)
- \( \chi^2_{\text{d.o.f}} (B_0^d) < 2.0 \)
- \( \cos(\theta) > 0.999 \)
- \( p_T(K^{*0}) > 3000 \) MeV
- \( \Delta M > 130 \) MeV

Table 5.6 shows the efficiency of the selection cuts on the sample where the baseline cuts were already applied.

### 5.2.5 Data Processing Algorithm Settings

In order to process the data into a format that can be easily worked with using ROOT software (c.f. Section 4.8.1), the Athena framework, described in Section 4.3 was used. Athena takes an AOD file (c.f. Section 4.7) and applies a user-defined algorithm to calculate and store the relevant event properties in a ROOT n-tuple format. The production of ROOT n-tuples used for the present \( B_0^d \to K^{*0}\mu^+\mu^- \) analysis...
5.2. EVENT SELECTION AND RECONSTRUCTION

### Table 5.6: Efficiency of the selection cuts (baseline cuts applied).

<table>
<thead>
<tr>
<th>Cut</th>
<th>Eff. on signal</th>
<th>Eff. on $b\bar{b} \rightarrow \mu^+\mu^- X$</th>
<th>Eff. on $c\bar{c} \rightarrow \mu^+\mu^- X$</th>
<th>Eff. on $B^0_d \rightarrow K^*\psi$</th>
<th>Eff. on $DY$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau/\Delta\tau(B^0_d) &gt; 12.75$</td>
<td>26.37%</td>
<td>6.45%</td>
<td>1.59%</td>
<td>40.75%</td>
<td>0.03%</td>
</tr>
<tr>
<td>$\chi^2$/n.d.f. ($B^0_d$) &lt; 2</td>
<td>58.88%</td>
<td>37.80%</td>
<td>48.34%</td>
<td>85.33%</td>
<td>52.77%</td>
</tr>
<tr>
<td>$\cos \theta &gt; 0.999$</td>
<td>30.93%</td>
<td>1.49%</td>
<td>4.98%</td>
<td>56.29%</td>
<td>1.96%</td>
</tr>
<tr>
<td>$p_T(K^*) &gt; 3000$ MeV</td>
<td>28.59%</td>
<td>4.55%</td>
<td>8.00%</td>
<td>70.04%</td>
<td>1.32%</td>
</tr>
<tr>
<td>$\Delta M &gt; 130$ MeV</td>
<td>87.25%</td>
<td>80.57%</td>
<td>91.53%</td>
<td>11.16%</td>
<td>89.01%</td>
</tr>
<tr>
<td>All cuts</td>
<td>7.65%</td>
<td>0.01%</td>
<td>0.005%</td>
<td>2.28%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

The algorithm searches for muon pair candidates that satisfy certain kinematic and geometric requirements. The selected muon tracks are then refitted to originate from the common vertex. The tracks that could not be identified as muons are used to reconstruct the $K^*$ vertex. The di-muon and $K^*$-candidates are then combined together to reconstruct the corresponding $B^0_d$-candidate. This way each event with a single di-muon candidate can contain multiple $B^0_d$-candidates. The common way to select the most probable $B^0_d$-candidate is to take the one with the smallest $\chi^2$/n.d.f. value for the reconstructed vertex.

The selection of the most probable $B^0_d$-candidate is done only after applying all other cuts.

### 5.2.6 Track Assignment

Since there is no dedicated detector for the hadron identification in ATLAS, the kaon and pion tracks cannot be distinguished. The track assignment is performed by testing two different mass hypotheses:

- 1st track is assigned the kaon invariant mass, 2nd track is assigned the pion invariant pion mass
- 2nd track is assigned the kaon invariant mass, 1st track is assigned the pion invariant pion mass
The hypothesis is taken that lead to the Kπ invariant mass closest to PDG value of the K*0 mass. The issue of correct track assignment is also discussed in Section 5.5.1 and Section 5.6.5.

5.3 Trigger Reweighting

The real data is selected by a certain combination of triggers (see Section 3.3 and Section 3.3.1). In order to compare the real and the simulated samples properly, a procedure of so called trigger re-weighting has to be applied to the used MC samples. Such a re-weighting only needs to take into account the most relevant triggers. The list of the most efficient triggers is obtained by looking at the corresponding entry in the data events after applying all the cuts and sorting them in terms of efficiency. However, the same event can be recorded by several correlated triggers. Therefore, the selection of the most efficient trigger signals is repeated in an iterative way. In each iteration the most efficient trigger is selected and a new event list is created, with the events fired by the most efficient trigger from the previous iteration being removed. Furthermore, some triggers had to be chosen for consistency reasons, since they are closely related to some of the most efficient ones. The list of the most efficient triggers estimated with the iterative procedure is shown in Table 5.7. The last four triggers there are selected additionally due to their related event topology. In total 1523 events with B0-d-candidates in the mass region between 5000 MeV and 5800 MeV were left after applying all the requirements, and 1457 of them (i.e. 95.7%) were picked by the 14 selected triggers. The remaining events were selected by various supporting triggers with high Prescale Factor (PS). Their efficiency would be hard to evaluate, so the corresponding events were removed from the analysis. A short description of each selected trigger is given in Appendix B.

The trigger PS can be significantly different during different data taking periods. To correctly account for this in the MC samples, the PSs for relevant trigger chains (i.e. listed in Table 5.7) were calculated with the ATLAS luminosity calculator [91]. The integrated luminosity and PSs obtained are listed in Table 5.8.

In the MC simulation two versions of trigger settings have been used in equal proportion of events. They corresponds to initial and tight L1 muon trigger configuration in data. To account for this fact the corresponding PS factors were divided by 2 (the last column in Table 5.8). In the analysis, each simulated event that passed one of the triggers from Table 5.8, was prescaled according to corresponding PS, i.e. the event was considered as passed the particular trigger with a probability 1/PSi.

Mass distribution of the simulated events satisfying all the cuts and selected by at least one of the 14 triggers listed in Table 5.8 is shown in Figure 5.7.
### 5.3. TRIGGER REWEIGHTING

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Most effective trigger</th>
<th>Events recorded</th>
<th>Events left</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>EF_mu4Tmu6_DiMu</td>
<td>445</td>
<td>1078</td>
</tr>
<tr>
<td>2</td>
<td>EF_2mu4_Bmumux</td>
<td>411</td>
<td>667</td>
</tr>
<tr>
<td>3</td>
<td>EF_mu18_MG_medium</td>
<td>212</td>
<td>455</td>
</tr>
<tr>
<td>4</td>
<td>EF_mu10_Jpsimumu</td>
<td>119</td>
<td>336</td>
</tr>
<tr>
<td>5</td>
<td>EF_2mu4_DiMu</td>
<td>71</td>
<td>265</td>
</tr>
<tr>
<td>6</td>
<td>EF_mu6_Jpsimumu_tight</td>
<td>69</td>
<td>196</td>
</tr>
<tr>
<td>7</td>
<td>EF_2mu4T_Bmumux</td>
<td>42</td>
<td>154</td>
</tr>
<tr>
<td>8</td>
<td>EF_mu4mu6_DiMu</td>
<td>28</td>
<td>126</td>
</tr>
<tr>
<td>9</td>
<td>EF_2mu10_loose</td>
<td>16</td>
<td>110</td>
</tr>
<tr>
<td>10</td>
<td>EF_mu18_L1J10</td>
<td>13</td>
<td>97</td>
</tr>
</tbody>
</table>

| EF\_mu6\_Jpsimumu | 10 | 87 |
| EF\_mu4\_DiMu     | 9  | 78 |
| EF\_mu18          | 7  | 71 |
| EF\_mu15\_mu10\_EFFF\_medium | 10 | 61 |

Table 5.7: List of the most effective triggers.

<table>
<thead>
<tr>
<th>Trigger name</th>
<th>Integrated luminosity(fb^{-1})</th>
<th>Luminosity-Weighted PS</th>
<th>Corr. factor</th>
<th>MC PS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>EF_mu4Tmu6_DiMu</td>
<td>2.481</td>
<td>1.989</td>
<td>.2</td>
</tr>
<tr>
<td>2</td>
<td>EF_2mu4_Bmumux</td>
<td>2.441</td>
<td>2.017</td>
<td>.2</td>
</tr>
<tr>
<td>3</td>
<td>EF_mu18_MG_medium</td>
<td>4.684</td>
<td>1.051</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>EF_mu10_Jpsimumu</td>
<td>2.763</td>
<td>1.782</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>EF_2mu4_DiMu</td>
<td>1.834</td>
<td>2.69</td>
<td>.2</td>
</tr>
<tr>
<td>6</td>
<td>EF_mu6_Jpsimumu_tight</td>
<td>1.069</td>
<td>4.614</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>EF_2mu4T_Bmumux</td>
<td>2.479</td>
<td>1.987</td>
<td>.2</td>
</tr>
<tr>
<td>8</td>
<td>EF_mu4mu6_DiMu</td>
<td>1.249</td>
<td>3.941</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>EF_2mu10_loose</td>
<td>4.920</td>
<td>1.001</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>EF_mu18_L1J10</td>
<td>3.530</td>
<td>1.395</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>EF_mu6_Jpsimumu</td>
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<td>8.848</td>
<td>.2</td>
</tr>
<tr>
<td>12</td>
<td>EF_mu4_DiMu</td>
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<td>70.44</td>
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</tr>
<tr>
<td>13</td>
<td>EF_mu18</td>
<td>2.776</td>
<td>1.774</td>
<td></td>
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<tr>
<td>14</td>
<td>EF_mu15_mu10_EFFF_medium</td>
<td>4.684</td>
<td>1.051</td>
<td></td>
</tr>
</tbody>
</table>

Table 5.8: Luminosity prescale corrected, luminosity weighted prescale and prescale used in MC analysis.
5.4 Fit Method

In order to obtain the values of the angular observables $A_{FB}$ (forward-backward asymmetry of the muons) and $F_L$ (fraction of the longitudinal polarization of $K^{*0}$) described in Section 2.8, events that survived all the cuts were split into several categories depending on the di-muon invariant mass $q^2$. The dependence of the angular observables on $K^+\pi^-$ invariant mass $p^2$ is neglected in this analysis, i.e. no S-wave is assumed.

The di-muon mass categories are usually called $q^2$-bins. $A_{FB}$ and $F_L$ are assumed to be constant within each bin, i.e. in the experiment only the averaged value of the observables is measured (c.f. Section 2.7). The binning repeats the one originally used by the Belle experiment [7]. The so called Belle bins are

- $0.045 \text{ GeV}^2 < q^2 < 2 \text{ GeV}^2$
- $2 \text{ GeV}^2 < q^2 < 4.3 \text{ GeV}^2$
- $4.3 \text{ GeV}^2 < q^2 < 8.68 \text{ GeV}^2$
- $10.09 \text{ GeV}^2 < q^2 < 12.86 \text{ GeV}^2$
- $14.18 \text{ GeV}^2 < q^2 < 16 \text{ GeV}^2$

Figure 5.7: Comparison between MC and data samples after baseline and selection cuts with imposed trigger requirements.
5.4. FIT METHOD

- $16 \text{ GeV}^2 < q^2 < 19 \text{ GeV}^2$
- $1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$

The two bins corresponding to the charmonium resonances ($8.68 \text{ GeV}^2 < q^2 < 10.09 \text{ GeV}^2$ and $12.86 \text{ GeV}^2 < q^2 < 14.18 \text{ GeV}^2$) are excluded from the analysis. The choice of the bin $1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$ is suggested in [92], where it is shown, that the theoretical predictions in this region have smaller uncertainties.

In the first bin, $0.045 \text{ GeV}^2 < q^2 < 2 \text{ GeV}^2$, the statistics of the data sample was too low to perform the angular analysis. In each of the remaining bins, a maximum likelihood fit to an unbinned multidimensional dataset is performed. The dimensions in this case consist of three variables: angles $\theta_K$ and $\theta_l$ described in Section 2.8 and invariant mass of $(K^+\pi^-\mu^+\mu^-)$-system. The latter is included in the fitted dataset to allow for the separation of the signal and background contributions, since they have a distinct shape of the invariant mass distribution. The fit is performed in two steps: first, the mass distribution alone is fitted as will be described in Section 5.4.1, next, the parameters of the mass distribution and the number of signal events are fixed to the values obtained in the first step and the multidimensional dataset is fitted. A similar sequential approach has been used by BaBar [8]. The fit models for the mass and angular distributions are discussed in the following sections.

5.4.1 Mass Model

The likelihood function for the fit of the $B^0_d$ candidates invariant mass distribution is defined by

$$L = \prod_{i=1}^{N} [N_{\text{sig}} \cdot M_{\text{sig}}(m_i, \delta_{m_i}) + N_{\text{bckg}} \cdot M_{\text{bckg}}(m_i)],$$

where $m_i$ is the measured invariant mass of $B^0_d$-candidates, $N_{\text{sig}}$ and $N_{\text{bckg}}$ denote the number of signal and background events, respectively, and $M_{\text{sig}}$ and $M_{\text{bckg}}$ are the probability density functions for the signal and background. Signal is modelled by a single Gaussian function with per-candidate mass errors $\delta_{m_i}$:

$$M_{\text{sig}}(m_i, \delta_{m_i}) = \frac{1}{\sqrt{2\pi s_m \delta_{m_i}}} \exp \left( \frac{-(m_i - m_{B^0_d})^2}{2(s_m \delta_{m_i})^2} \right).$$

The per-candidate mass errors are obtained from the covariance matrix provided by the output of the vertex reconstruction algorithm. If these errors are under- or over-estimated, it would be reflected in the value of the scale parameter $s_m$. Ideally, $s_m$ should be very close to one.

The background description is provided by an exponential function

$$M_{\text{bckg}}(m_i) = e^{-\lambda \cdot m_i}.$$
In Figure 5.8 the fitted distribution of \((K^+\pi^-\mu^+\mu^-)\) invariant mass over the full range of \(q^2\) is shown. This way, in the \(B^0_d\) mass range between 4900 MeV and 5700 MeV in total 466 signal events were found in the available data sample.

5.4.2 Angular Model

In experiment, the number of events in some bins may be too low to perform a full fit to \( \frac{d^2\Gamma}{dq^2 d\cos \theta_l d\cos \theta_K d\phi} \). Therefore the reduced angular distributions (see Eq. 2.40 and Eq. 2.41) are used to describe the shape of \( \cos \theta_l \) and \( \cos \theta_K \) of the signal events in each \( q^2 \) bin:

\[
A_{l,sig}(\cos \theta_{l,i}) = \frac{3}{4} F_L \left( 1 - \cos^2 \theta_{l,i} \right) + \frac{3}{8} \left( 1 - F_L \right) \left( 1 + \cos^2 \theta_{l,i} \right) + A_{FB} \cos \theta_{l,i}
\]

\[
A_{K,sig}(\cos \theta_{K,i}) = \frac{3}{2} F_L \cos^2 \theta_{K,i} + \frac{3}{4} \left( 1 - F_L \right) \left( 1 - \cos^2 \theta_{K,i} \right)
\]

Here \( A_{FB} \) and \( F_L \) are assumed to be constant within each bin.

Since a decay rate cannot be negative, certain combinations of \( A_{FB} \) and \( F_L \) values are unphysical. The region of physically allowed values in the \( A_{FB} - F'_L \) space is defined by the constraint

\[
|A_{FB}| \leq \frac{3}{4} \left( 1 - F_L \right)
\]

Figure 5.8: Fit of the \( B^0_d \) -candidates mass distribution in data over the full \( q^2 \) range.
that can be obtained by requiring $\frac{d\Gamma}{dq^2 d\cos\theta_l} > 0$.

The angular shape of the background events was described by the 2nd order Chebyshev polynomials in both $\cos\theta_l$ and $\cos\theta_K$:

$$A_{L,bckg}(\cos \theta_{L,i}) = 1 + p_1 \cos \theta_{L,i} + p_2 (2(\cos \theta_{L,i})^2 - 1)$$  \hspace{1cm} (5.23)

$$A_{K,bckg}(\cos \theta_{K,i}) = 1 + p_1 \cos \theta_{K,i} + p_2 (2(\cos \theta_{K,i})^2 - 1)$$

A sufficient agreement with the data is achieved with the simple polynomial model, despite the fact that it does not intend to properly describe the shape of each background component.

5.4.3 Acceptance Maps

Due to the detector shape and material distribution (see Figure 3.5, 3.9, 3.10), the efficiency of particle detection depends on its direction of flight. The efficiency of the applied cuts can also depend on the phase space region and therefore “re-shape” the true angular distributions. This effect can be accounted for by determining the so called acceptance maps, i.e. functions describing the efficiency of the detector and the applied cuts for each value of $\cos \theta_l$ and $\cos \theta_K$.

To obtain the acceptance maps, full detector simulation is applied to special MC samples of the $B^0_d \rightarrow K^0 \mu^+ \mu^-$ events. This sample was generated with flat angular distributions in $\cos \theta_l$ and $\cos \theta_K$, so any angular shape different from flat is due to detector effects and applied cuts, mainly on $p_T$ and $|\eta|$ of the tracks. The trigger re-weighting procedure described in Section 5.3 was also applied to this “acceptance sample”.

In each $q^2$-bin the resulting distributions of $\cos \theta_L$ and $\cos \theta_K$ are fitted by fifth order polynomials:

$$f(\cos \theta_l) = a_1 + a_2 \cos \theta_l + a_3 \cos^2 \theta_l + a_4 \cos^3 \theta_l + a_5 \cos^4 \theta_l + a_6 \cos^5 \theta_l$$

$$f(\cos \theta_K) = b_1 + b_2 \cos \theta_K + b_3 \cos^2 \theta_K + b_4 \cos^3 \theta_K + b_5 \cos^4 \theta_K + b_6 \cos^5 \theta_K$$  \hspace{1cm} (5.24)

Here it is assumed that the acceptances for $\cos \theta_l$ and $\cos \theta_K$ are uncorrelated. The potential bias in the fit results due to this assumption will be discussed in Section 5.6.8.

The acceptance maps for the different $q^2$-bins are shown in Appendix C. In order to obtain the correct values of the angular observables from the fit, probability density functions describing the angular distribution of the signal (Eq. 5.20 and Eq. 5.20) have to be multiplied with the corresponding acceptance functions, as described in the next section.
5.5 Full Fit Model

Finally, the unbinned maximum likelihood fit to the multidimensional dataset is performed with the likelihood function defined as

\[
\mathcal{L} = \prod_{i=1}^{N} \left[ N_{\text{fix}}^{\text{sig}} \cdot M_{\text{sig}}(m_{i}, \delta_{m_{i}}|\text{fixed}) \cdot (A_{L,\text{sig}}(\cos \theta_{L,i}) \cdot f(\cos \theta_{1})) \cdot (A_{K,\text{sig}}(\cos \theta_{K,i}) \cdot f(\cos \theta_{K})) \right] + N_{\text{bckg}} \cdot M_{\text{bckg}}(m_{i}|\text{fixed}) \cdot (A_{L,\text{bckg}}(\cos \theta_{L,i}) \cdot A_{K,\text{bckg}}(\cos \theta_{K,i})).
\]

(5.25)

Here \(M_{\text{sig}}(m_{i}, \delta_{m_{i}}|\text{fixed})\) and \(M_{\text{bckg}}(m_{i}|\text{fixed})\) are probability density functions for mass distribution of signal and background, defined in Eq. 5.18 and Eq. 5.19, respectively. At this stage, the number of signal events \(N_{\text{sig}}\), the mean of the Gaussian \(m_{B_{0}d}\), the scale factor \(s_{m}\) and the exponent \(\lambda\) describing the background contribution, are fixed at the values obtained from the fit discussed in Section 5.4.1. Signal probability density functions for the angular distributions are modelled according to Eq. 5.20 and Eq. 5.21 and are multiplied by the acceptance functions \(f(\cos \theta_{K})\) and \(f(\cos \theta_{1})\) defined in Eq. 5.24, assuming that the acceptances for \(\cos \theta_{1}\) and \(\cos \theta_{K}\) are uncorrelated. The angular distributions of the background events are described in Eq. 5.23. They do not have to be multiplied by the acceptance maps, since the background description is very general and one is not interested in the real parameters of the background shape in this analysis.

A remark has to be made regarding the dimensionality of the fit model. The fit is performed on the multidimensional dataset, and the fit function includes three dimensions (mass and two angles). However, it is constructed as a product of one-dimensional functions and, therefore, does not take into account the potential correlations between the variables. In this sense, the fit model will be referred to as "one-dimensional" when the systematic effects are discussed (Section 5.6.10).

5.5.1 Kaon-Pion Misidentification Effect

Since the ATLAS detector cannot distinguish between kaons and pions, the track assignment based on "closest \(K^{*0}\) mass" hypothesis (see Section 5.2.6) leads to a non-negligible misidentification rate. In the MC sample of the simulated signal events, the truth information of the track is available, so one can obtain the misidentification probability by comparing the result of "closest \(K^{*0}\) mass" assignment with the truth information. This way it was found, that in approximately \(\eta = 12.5\%) of the events that survived all the cuts, kaon and pion tracks are swapped.

Since the angle \(\theta_{K}\) is defined with respect to the kaon track (see Section 2.7), this means that in 12.5% of events angle \((\pi - \theta_{K})\) is taken instead of \(\theta_{K}\). This has no direct effect on the measurement, since this angle enters the distributions of Eq. 2.40 and Eq. 2.41 as \(\cos^{2} \theta_{K}\). However, in this case \(B\) and \(\bar{B}\)-mesons
are also misidentified. As a result, the angle $\theta_l$ is defined with respect to a wrong muon track, so in fact $(\pi - \theta_l)$ is taken. Since it enters the Eq. 2.40 as $A_{FB}\cos\theta_l$, this leads to taking a wrong sign of the $A_{FB}$. Effectively the total number of signal events consists of two terms: events with correctly identified hadronic tracks and events with kaon and pion tracks swapped

$$N_{\text{sig}} = N_{\text{corr}} A_{FB} - N_{\text{misid}} A_{FB} = (1 - \eta) N_{\text{sig}} A_{FB} - \eta N_{\text{sig}} A_{FB} = (1 - 2\eta) N_{\text{sig}} A_{FB}, \quad (5.26)$$

if $\eta = 0.125$ is the misidentification probability. Thus, the fitted value of $A_{FB}$ has to be divided by $(1 - 2\eta)$ to obtain the real value. The values of the forward-backward asymmetry listed in Table 6.1 are obtained by calculating

$$A_{FB} = \frac{A_{FB}^{\text{fitted}}}{(1 - 2\eta)}. \quad (5.27)$$

5.6 Systematic Uncertainties

In every physics analysis, the choice of the analysis method and the assumptions or simplifications made at the various stages, may lead to a bias in the obtained results. A careful investigation of the potential sources of the bias and an estimation of the size of the systematic effects they induce, is a very important task of the analysis. In this section, the various sources of systematic uncertainties, considered in this analysis, are discussed. They include the choice of the fit range, model and procedure, acceptance effects, correlations, contributions of other decays, also misreconstructed ones, and the uncertainties due to the limited size of the MC samples used for studies. The total systematic uncertainty is calculated as

$$\sigma_{\text{sys,tot}} = \sqrt{\sum_i \sigma_{\text{sys},i}^2}. \quad (5.28)$$

5.6.1 Sequential Fit

As it was discussed in Section 5.4, the fit procedure consists of two steps: first the mass distribution is fitted, then its parameters are fixed, and the full multidimensional dataset is fitted with a likelihood function presented in Eq. 5.25. This approach had to be used mainly due to the low statistics in the low $q^2$-bin $2 \text{ GeV}^2 < q^2 < 4.3 \text{ GeV}^2$. If all the parameters were left free, their extraction from a one-step fit was not possible. For consistency reasons, the same sequential fit procedure was used in the other bins. However, the statistics in this case was sufficient to also perform a fit without fixing the parameters of the mass distribution, and thus estimate the systematic uncertainty on the value of the angular parameters. The uncertainty is taken as a difference between the values obtained from the sequential fit and from the
one-step fit, where all the parameters are left free. In the $2 \text{ GeV}^2 < q^2 < 4.3 \text{ GeV}^2$ bin, the simultaneous fit resulted in an unrealistically large value of the scale parameter of the signal Gaussian (see Eq. 5.18).

In this specific bin, the systematic uncertainty was assigned in the following way. Taking the sequential fit approach, the scale factor and its error were used to constrain the scale factor in the simultaneous fit. Performing the simultaneous fit with this Gaussian constraint, the resulting values for $A_{FB}$ and $F_L$ were compared to the actually measured values, and the deviation was taken as the systematic uncertainty.

In all bins of $q^2$, the error of the variables $A_{FB}$ and $F_L$ are of the same order, see Table 5.9. This shows, that the treatment of the statistical uncertainty is not biased by using a sequential approach (i.e. by neglecting potential correlations between mass and angular variables). Furthermore, also the values of $A_{FB}$ and $F_L$ are compatible between the two approaches, except in the lowest $q^2$ bin, where this is found to be the dominating source of the systematic uncertainty.

### 5.6.2 Mass and Angular Fit Model

Systematic uncertainties of the mass fit model on the angular observables have been tested by varying the mass background probability density function shape between exponential, second and third order polynomials. For each shape, the fit on angular observables was performed with fixed mass parameters. No effect was observed on the number of fitted signal events and consequently $F_L$ and $A_{FB}$ values due to the change of mass background shape.

The angular background shape was changed between $2^{\text{nd}}$ and $3^{\text{rd}}$ order Chebyshev polynomials for $\cos \theta_L$ and $\cos \theta_K$ independently to study possible effects on the measured values of $F_L$ and $A_{FB}$. The impact of angular background model is rather small, up to 2.5% on $A_{FB}$ and 7% on $F_L$, depending on the $q^2$-bin. The results are summarized in Table 5.10 and 5.11.

---

<table>
<thead>
<tr>
<th>$q^2$ range (GeV$^2$)</th>
<th>$A_{FB}^{\text{eq}}$</th>
<th>$A_{FB}^{\text{mult.}}$</th>
<th>$F_L^{\text{eq}}$</th>
<th>$F_L^{\text{mult.}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.00 \leq q^2 &lt; 4.30$</td>
<td>$0.218 \pm 0.276$</td>
<td>$0.372 \pm 0.293$</td>
<td>$0.257 \pm 0.184$</td>
<td>$0.328 \pm 0.196$</td>
</tr>
<tr>
<td>$4.30 \leq q^2 &lt; 8.68$</td>
<td>$0.242 \pm 0.128$</td>
<td>$0.238 \pm 0.128$</td>
<td>$0.366 \pm 0.108$</td>
<td>$0.369 \pm 0.108$</td>
</tr>
<tr>
<td>$10.09 \leq q^2 &lt; 12.86$</td>
<td>$0.086 \pm 0.092$</td>
<td>$0.080 \pm 0.085$</td>
<td>$0.497 \pm 0.092$</td>
<td>$0.506 \pm 0.095$</td>
</tr>
<tr>
<td>$14.18 \leq q^2 &lt; 16.00$</td>
<td>$0.434 \pm 0.180$</td>
<td>$0.443 \pm 0.192$</td>
<td>$0.293 \pm 0.146$</td>
<td>$0.291 \pm 0.147$</td>
</tr>
<tr>
<td>$16.00 \leq q^2 &lt; 19.00$</td>
<td>$0.156 \pm 0.098$</td>
<td>$0.155 \pm 0.098$</td>
<td>$0.348 \pm 0.078$</td>
<td>$0.344 \pm 0.079$</td>
</tr>
<tr>
<td>$1.00 \leq q^2 &lt; 6.00$</td>
<td>$0.070 \pm 0.204$</td>
<td>$0.065 \pm 0.205$</td>
<td>$0.178 \pm 0.150$</td>
<td>$0.174 \pm 0.154$</td>
</tr>
</tbody>
</table>

Table 5.9: Results on the angular variables $A_{FB}$ and $F_L$ using the sequential approach (mass fit, fix parameters and perform angular fit) and using a simultaneous approach (leave mass parameters free and do the fit in one step).
5.6. SYSTEMATIC UNCERTAINTIES

### Table 5.10: Systematic uncertainties on $A_{FB}$ in bins of $q^2$ due to the mass fit region, angular fit models, $B^\pm \to K^\pm \mu^+ \mu^-$ contamination, acceptance maps and sequential fit approach.

<table>
<thead>
<tr>
<th>$q^2$ range (GeV$^2$)</th>
<th>fit region</th>
<th>ang. fit</th>
<th>$B^\pm \to K^\pm \mu^+ \mu^-$</th>
<th>acc. maps</th>
<th>fit</th>
<th>SUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.00 $\leq q^2 &lt; 4.30$</td>
<td>0.02</td>
<td>0.01</td>
<td>0.08</td>
<td>0.01</td>
<td>0.10</td>
<td>0.136</td>
</tr>
<tr>
<td>4.30 $\leq q^2 &lt; 8.68$</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.013</td>
</tr>
<tr>
<td>10.09 $\leq q^2 &lt; 12.86$</td>
<td>0.03</td>
<td>0.01</td>
<td>0.02</td>
<td>0.00</td>
<td>0.031</td>
<td></td>
</tr>
<tr>
<td>14.18 $\leq q^2 &lt; 16.00$</td>
<td>0.01</td>
<td>0.01</td>
<td>0.03</td>
<td>0.02</td>
<td>0.039</td>
<td></td>
</tr>
<tr>
<td>16.00 $\leq q^2 &lt; 19.00$</td>
<td>0.02</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
<td>0.026</td>
<td></td>
</tr>
<tr>
<td>1.00 $\leq q^2 &lt; 6.00$</td>
<td>0.05</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.069</td>
<td></td>
</tr>
</tbody>
</table>

### Table 5.11: Systematic uncertainties on $F_L$ in bins of $q^2$ due to the mass fit region, angular fit models, $B^\pm \to K^\pm \mu^+ \mu^-$ contamination, acceptance maps and sequential fit approach.

<table>
<thead>
<tr>
<th>$q^2$ range (GeV$^2$)</th>
<th>fit region</th>
<th>ang. fit</th>
<th>$B^\pm \to K^\pm \mu^+ \mu^-$</th>
<th>acc. maps</th>
<th>fit</th>
<th>SUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.00 $\leq q^2 &lt; 4.30$</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
<td>0.05</td>
<td>0.058</td>
</tr>
<tr>
<td>4.30 $\leq q^2 &lt; 8.68$</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
<td>0.021</td>
<td></td>
</tr>
<tr>
<td>10.09 $\leq q^2 &lt; 12.86$</td>
<td>0.04</td>
<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
<td>0.042</td>
<td></td>
</tr>
<tr>
<td>14.18 $\leq q^2 &lt; 16.00$</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
<td>0.025</td>
<td></td>
</tr>
<tr>
<td>16.00 $\leq q^2 &lt; 19.00$</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.023</td>
<td></td>
</tr>
<tr>
<td>1.00 $\leq q^2 &lt; 6.00$</td>
<td>0.02</td>
<td>0.01</td>
<td>0.00</td>
<td>0.03</td>
<td>0.034</td>
<td></td>
</tr>
</tbody>
</table>

5.6.3 Mass Fit Range

There are two potential issues connected with the choice of the fit region in $B^0_d$ candidates mass distribution. First, the fitted number of signal events may change depending on the fit region, and affect the angular distributions since it enters the likelihood model in Eq. 5.25. Second, in Figure 5.5 it can be seen that due to the $\Delta M$ cut the $B^0_d$ mass sidebands are not fully covered in all $q^2$-bins, i.e. this cut results in a certain shaping of the background component, especially in the central bin. The choice of the mass fit region therefore may affect the shape of the background.

The first problem was investigated by performing a mass fit to the data in full $q^2$ range in various ranges of $B^0_d$ mass between 4600 MeV and 6000 MeV and looking at the number of fitted signal events. It was shown that this number varies within its statistical uncertainty. The results of this study are shown in Table 5.12. To deal with the second issue, the mass fit region had to be chosen individually in each $q^2$-bin in the way, that the background shape is the least affected by the $\Delta M$ cut. The fit regions chosen as default in each bin are listed in Table 5.13. In each bin, for all $B^0_d$ mass ranges, the same procedure was
Followed: obtain the parameters of the mass distribution from the fit, fix them, then perform the full fit to obtain the angular parameters. The maximum difference between the values of $F_L$ and $A_{FB}$ obtained from the fit in the different ranges of $B_d^0$ invariant mass was taken as the systematic uncertainty. The results are summarized in Table 5.10 and 5.11. Systematic uncertainty due to the variation of the mass has the largest impact in the central bin, since the amount of events in the sidebands in this region is limited due to the $\Delta M$ cut.

### 5.6.4 Fit Bias

To test the performance of the fit model and check for a possible bias in the fitted values of the angular observables, a toy MC study was performed. In this study, a multidimensional dataset containing mass, $\cos\theta_l$ and $\cos\theta_K$ was generated according to the model in Eq. 5.25, but without acceptance functions. Values of all the parameters at the generation step were taken randomly within some realistic range. This
5.6. SYSTEMATIC UNCERTAINTIES

Figure 5.9: Pull distributions of $A_{FB}$ (left) and $F_L$ (right) observables, obtained from a toy MC study.

dataset was then fitted in the same way as the real data, following the sequential procedure described in Section 5.4, but also without the acceptance functions. The procedure was repeated 10000 times, and the pull distributions of $A_{FB}$ and $F_L$ were obtained. The pull value of the $a$ observable is calculated as

$$a_p = \frac{a_{\text{gen}} - a_{\text{fit}}}{\delta a},$$

(5.28)

where $\delta a$ is the uncertainty of the fitted value. If the fit is not biased, the pull value should follow the standard normal distribution. The deviation of the pull distribution mean value from zero gives the size of the bias. The deviation of its width from one shows whether the parameter errors are under- or overestimated. The resulting pull distributions of $A_{FB}$ and $F_L$ are shown in Figure 5.9. The following values were obtained for the pull mean and width for $A_{FB}$ and $F_L$:

$$\langle A_{FB} \rangle_{\text{pull}}^{\text{mean}} = 0.001 \pm 0.009,$$
$$\langle A_{FB} \rangle_{\text{pull}}^{\text{σ}} = 0.011 \pm 0.008,$$

(5.29)

$$\langle F_L \rangle_{\text{pull}}^{\text{mean}} = 0.066 \pm 0.010,$$
$$\langle F_L \rangle_{\text{pull}}^{\text{σ}} = 1.004 \pm 0.008.$$  

They are consistent with those of the standard normal distribution, therefore, no systematic uncertainty was assigned due to the potential bias of the fit model.

5.6.5 Kaon-Pion Misidentification

The effect of misidentification of the kaon and pion tracks was discussed in Section 5.5.1. It was accounted for by introducing a multiplicative coefficient $\eta$ to the fitted value of $A_{FB}$ and its uncertainty. Due to the limited size of the signal MC sample used to estimate the fraction of misreconstructed events,
the obtained $\eta$ value has a statistical uncertainty of less than 1%. No significant systematic uncertainty is introduced by this effect.

### 5.6.6 S-Wave

Although in Section 2.9 it is discussed, that the decay $B \rightarrow K_s^0(\rightarrow K\pi)\mu^+\mu^-$ via an intermediate scalar state $K_s^0$, called S-wave, can have a strong influence on the angular observables, in practice it becomes a non-trivial task to separate the S-wave contribution, especially in the relatively small datasets. In principle, there are several ways to estimate the S-wave shape and the size of its contribution. The easiest one would be to take its parameters from a MC simulated sample. It is, however, not possible to generate such a sample, since neither the branching fraction, nor the angular parameters of the S-wave are known. Another approach would be to include the S-wave contribution in the fit model, i.e. use Eq. 2.42 for the signal probability density function. That would require a higher number of events in the dataset in order to perform a fit with additional free parameters. Next, one could extract the S-wave contribution from the sidebands of the $B^0_d$ and $K^*0$ mass distribution, and interpolate it over the signal region. Again, this would require larger statistics, especially in the right sideband of the $B^0_d$ mass.

In this analysis, a relatively simplified approach was used, which nevertheless allows for the estimation of the systematic uncertainty due to the S-wave. The angular shape of all background is not modelled but rather fitted by a very general polynomial shape. Consequently it can be assumed that the angular distribution of the S-wave follows the background shape. In the $B^0_d$ candidates mass distribution, the S-wave behaves like a signal decay, since it has the same final state. Therefore, to check for the potential S-wave influence on the angular observables, an additional term was included in the fit model, which follows the signal shape in the $B^0_d$ mass distribution (see Eq. 5.18) and the background shape in the angular distributions (Eq. 5.20 and Eq. 5.21). The size of this contribution was varied up to 30% of the number of signal events. Only if the S-wave fraction reaches 30% of the number of signal events, a slight effect on the $A_{FB}$ and $F_L$ fitted values starts to be seen. With a more realistic contribution of 8%, as estimated by BaBar [52], no effect is observed, therefore, no systematic uncertainty is assigned due to the S-wave presence.

### 5.6.7 Misreconstructed Decay $B^\pm \rightarrow K^\pm \mu^+\mu^-$

The decay $B^\pm \rightarrow K^\pm \mu^+\mu^-$ with one additional hadronic track may resemble the final state of the signal channel $B^0_d \rightarrow K^*0\mu^+\mu^-$. As discussed in the end of Section 5.2.3, a cut was applied to remove the events that may have been misreconstructed: the di-muon pairs were combined with each of the charged hadronic tracks (reconstructed as kaon or pion) under the hypothesis that the three tracks originate
from $B^\pm$ decay, and the events with $5229\,\text{MeV} < m(h\mu^+\mu^-) < 5329\,\text{MeV}$, where $h$ denotes a hadronic track, were removed. The difference between the values of the angular observables coming from the fit to the data sample with and without misreconstructed $B^\pm$ candidates was taken as a systematic uncertainty, shown in Table 5.11 and Table 5.10. The highest contribution is found in the lowest $q^2$-bin $2.00\,\text{GeV}^2 < q^2 < 4.30\,\text{GeV}^2$ due to the low statistics in this bin. The results quoted in Table 6.1 are obtained from the fit to the data sample where no $B^\pm$ candidates are removed.

5.6.8 Acceptance Maps

Three kinds of sources coming from the acceptance corrections discussed in Section 5.4.3 may lead to systematic effects on the angular observables. First, it is the limited size of the MC sample used to determine the shape of the acceptance functions, second, it is the model used to fit this shape, and third is the effect of correlation between the two angles. The systematic uncertainty due to acceptance maps shown in Table 5.10 and Table 5.11 is obtained by adding up the three sources in quadrature.

The main contribution (more than 50%, depending on the $q^2$-bin) to this systematic uncertainty arises from the limited statistics of the signal events sample used for the creation of the angular acceptance maps. The total number of events in this sample after all the cuts, in the full range of $q^2\,0.04\,\text{GeV}^2 < q^2 < 19.00\,\text{GeV}^2$, equals 42504.

The angular distributions in $\cos\theta_L$ and $\cos\theta_K$ used to determine the "sculpting" of the true shape by the detector and the cuts are fitted with fifth order polynomials (Eq. 5.24). To study the effect of the acceptance model, a fit with symmetric acceptances was performed. Additionally, signal MC samples were generated that reproduced the angular distributions predicted by the SM [46] and measured in the LHCb experiment [12], instead of the default “flat” shapes.

Finally, the acceptance model assumes no correlation between the two angles $\theta_L$ and $\theta_K$. To justify this, the correlation coefficient $r$ was calculated as

$$r = \frac{\sum_{i=1}^{N} (\cos\theta_L - \langle \cos\theta_L \rangle)(\cos\theta_K - \langle \cos\theta_K \rangle)}{\sqrt{\sum_{i=1}^{N} (\cos\theta_L - \langle \cos\theta_L \rangle)^2 \sum_{i=1}^{N} (\cos\theta_K - \langle \cos\theta_K \rangle)^2}}, \tag{5.30}$$

where $N = 42504$ is the total number of events in the sample, $\langle \cos\theta_L \rangle = \sum_{i=1}^{N} \cos(\theta_L)_i/N$, $\langle \cos\theta_K \rangle = \sum_{i=1}^{N} \cos(\theta_K)_i/N$ and index $i$ refers to each individual event. The obtained value $r = -0.0036$ indicates a very small correlation between the distribution of the two angles, which can be neglected when con-
Figure 5.10: Scatter plot of \( \cos \theta_K \) and \( \cos \theta_l \) in the signal MC sample used to obtain the acceptance maps.

The acceptance maps are constructed, i.e. the acceptance in \( \theta_l \) and \( \theta_K \) can be obtained independently. Figure 5.10 presents the correlation plot of the two angles \( \theta_l \) and \( \theta_K \) in the signal MC sample used to obtain the acceptance maps.

5.6.9 Angular Correlations

A similar question arises for the fit model itself. When one-dimensional distributions (Eq. 5.20 and Eq. 5.21) are used for the fit, potential correlations between the two angles are not taken into account. To study the validity of this simplification, the correlation coefficient \( r \) (Eq. 5.30) was calculated for the angles \( \theta_K \) and \( \theta_l \) in the real data and was found to be \( r = -0.031 \), so the assumption of uncorrelated angles is valid. The corresponding correlation plot is shown in Figure 5.11. The total number of events in this data sample over the full \( q^2 \) range \( 0.04 \text{ GeV}^2 < q^2 < 19.00 \text{ GeV}^2 \), in the range of \( B_d^0 \) candidates mass \( 4600 \text{ MeV} < m(K^+\pi^-\mu^+\mu^-) < 6000 \text{ MeV} \) consists of 4466 events.

5.6.10 Dimensionality of the Model

Following the remark in Section 5.5, another cross-check of the one-dimensional model validity was performed with a toy MC study. In this study, a three-dimensional dataset (mass, angles \( \theta_l \) and \( \theta_K \)) was generated according to the following model:

\[
\mathcal{F} = \mathcal{M}(m) \cdot \mathcal{A}(\cos \theta_K, \cos \theta_l), \quad (5.31)
\]
where the mass model $M(m)$ is constructed in the same way as described in Section 5.4.1, and the angular model $A(\cos\theta_K, \cos\theta_l)$ follows Eq. 2.39. The background events were generated without any changes in the model. The dataset was then fitted with the usual one-dimensional model (Eq. 5.25) without acceptance functions. The procedure was repeated 10000 times, and the pull distributions for $A_{FB}$ and $F_L$ were obtained, with the pull value calculated as in Eq. 5.28. The pull distributions are shown in Figure 5.12. The mean and the width of the pull distributions are

$$\langle A_{FB} \rangle^{\text{pull}}_{\text{mean}} = 0.005 \pm 0.009, \quad \langle A_{FB} \rangle^{\text{pull}}_{\sigma} = 0.919 \pm 0.007,$$

$$\langle F_L \rangle^{\text{pull}}_{\text{mean}} = 0.09 \pm 0.01, \quad \langle F_L \rangle^{\text{pull}}_{\sigma} = 0.983 \pm 0.008.$$

No significant bias is observed, consequently, no systematic uncertainty is assigned due to dimensionality of the fit model.
Figure 5.12: Pull distributions of $A_{FB}$ (left) and $F_L$ (right) for the toy datasets generated with a 2D-model and fitted with a 1D-model.
Chapter 6

Results and Conclusions

In this chapter, the results of the measurement described in Chapter 5 are presented, including the uncertainties on the obtained values. Comparison with the results of other experiments is made in Section 6.2, and an outlook is given in Section 6.3.

6.1 Results

The final results of the angular analysis of the $B^0_d \rightarrow K^{*0} \mu^+ \mu^-$ decay described in this thesis are presented in Table 6.1. In each $q^2$ bin, the total number of signal events with its statistical uncertainty and the measured values of the forward-backward asymmetry of muons $A_{FB}$ and the fraction of longitudinal polarization of the $K^{*0}$ meson, together with their statistical and systematic uncertainties, are presented. The first number in the table shows the measured value, the second its statistical uncertainty, and the third the total systematic uncertainty. Statistical uncertainty dominates in every $q^2$ bin. The systematic uncertainties of $A_{FB}$ and $F_L$ reach up to 50% of the statistical ones, depending on the $q^2$-bin. The total uncertainty $\sigma_{tot} = \sqrt{\sigma_{stat}^2 + \sigma_{sys}^2}$ is 112% of the corresponding statistical uncertainty, at most.

In Figure 6.1 and Figure 6.2 the comparison of the measured values with the SM predictions is shown for $A_{FB}$ and $F_L$, respectively. Theoretical predictions are calculated with the "EOS" software [94, 95], originally developed at TU Dortmund. The width of the light green band corresponds to the theoretical uncertainty. The dark green bars show the theoretical prediction averaged over each $q^2$-bin, the horizontal size of the bar corresponds to the bin size, and the vertical shows the theoretical uncertainty on the averaged value. No theoretical prediction exists for the central $q^2$ bin between the two charmonium resonances, since the techniques used for this calculation provide reliable results in the limit of either low or large values of $q^2$. The measured quantities are shown as black crosses, with vertical line showing the total uncertainty of the measurement $\sigma_{tot}$. In the forward-backward asymmetry plot no deviation
Table 6.1: Summary of the fit results for the different bins of $q^2$. Number of signal events $N_{\text{sig}}$ from the mass fit and its statistical uncertainty, forward-backward asymmetry $A_{\text{FB}}$ and longitudinal polarization $F_L$ including statistical and systematic uncertainties [6, 93].

<table>
<thead>
<tr>
<th>$q^2$ range (GeV$^2$)</th>
<th>$N_{\text{sig}}$</th>
<th>$A_{\text{FB}}$</th>
<th>$F_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.00 &lt; q^2 &lt; 4.30$</td>
<td>$19 \pm 8$</td>
<td>$0.22 \pm 0.28 \pm 0.14$</td>
<td>$0.26 \pm 0.18 \pm 0.06$</td>
</tr>
<tr>
<td>$4.30 &lt; q^2 &lt; 8.68$</td>
<td>$88 \pm 17$</td>
<td>$0.24 \pm 0.13 \pm 0.01$</td>
<td>$0.37 \pm 0.11 \pm 0.02$</td>
</tr>
<tr>
<td>$10.09 &lt; q^2 &lt; 12.86$</td>
<td>$138 \pm 31$</td>
<td>$0.09 \pm 0.09 \pm 0.03$</td>
<td>$0.50 \pm 0.09 \pm 0.04$</td>
</tr>
<tr>
<td>$14.18 &lt; q^2 &lt; 16.00$</td>
<td>$32 \pm 14$</td>
<td>$0.48 \pm 0.19 \pm 0.05$</td>
<td>$0.28 \pm 0.16 \pm 0.03$</td>
</tr>
<tr>
<td>$16.00 &lt; q^2 &lt; 19.00$</td>
<td>$149 \pm 24$</td>
<td>$0.16 \pm 0.10 \pm 0.03$</td>
<td>$0.35 \pm 0.08 \pm 0.02$</td>
</tr>
<tr>
<td>$1.00 &lt; q^2 &lt; 6.00$</td>
<td>$42 \pm 11$</td>
<td>$0.07 \pm 0.20 \pm 0.07$</td>
<td>$0.18 \pm 0.15 \pm 0.03$</td>
</tr>
</tbody>
</table>

Figure 6.1: Forward-backward asymmetry $A_{\text{FB}}$, including statistical and systematic uncertainties, compared to theoretical predictions calculated for the limits of small values of $q^2$ and large values of $q^2$ including theoretical uncertainties [6, 93].
6.2. COMPARISON WITH OTHER EXPERIMENTS

In Figure 6.3 and Figure 6.4 the comparison of ATLAS results with the results from LHCb [12], CMS [11], CDF [10], BaBar [9] and Belle [7] experiments is shown. Same as when comparing the result with the theoretical prediction, if 3σ is taken as a measure of deviation, the results from all experiments are in agreement. It is interesting to see that three of the experiments (ATLAS, CDF and BaBar) have measured a value of $F_L$ in the second $q^2$-bin that is about 2.5 times lower than the one predicted in the SM. It
remains yet to see whether in the coming analysis of the full 2012 datasets by the LHC experiments this difference will remain.

Currently the most precise measurements of the $B^0_d \to K^{*0} \mu^+ \mu^-$ branching fraction and the angular observables in the low $q^2$ region are performed by the LHCb experiment [12]. The main reasons for that are the beam and trigger settings provided in this experiment, which improve the signal over background ratio and allow to gain higher statistics in the low $q^2$ region. The LHCb analysis is based on 1 fb$^{-1}$ of data. A big advantage of this experiment is excellent separation between kaon and pion tracks, provided by the Ring Imaging Cherenkov (RICH) detectors [96].

The ATLAS trigger settings strongly reduce the amount of data in the low $q^2$ bins, but in the high $q^2$ region the uncertainty of the measurement is of the same size as at LHCb. Higher integrated luminosity available at ATLAS (20.3 fb$^{-1}$ [97], compared to 2.08 fb$^{-1}$ recorded by LHCb [98]) will play an important role in the analysis of the 2012 data. ATLAS and CMS results are also complementary to the ones of LHCb, since the latter experiment performs the measurement in a different region of pseudorapidity ($2 < |\eta| < 5$, compared to $|\eta| < 2.5$ in ATLAS and $|\eta| < 2.4$ in CMS).
6.3. OUTLOOK

The work on the analysis of 20.3 fb$^{-1}$ [97] of the data collected by ATLAS in the year 2012 is currently ongoing. The author of this thesis and her colleagues are hoping to obtain the updated results of the $B^0_d \to K^{*0} \mu^+ \mu^-$ angular analysis and perform the branching ratio measurement by the end of 2014.

It is already possible to estimate the precision of the $A_{FB}$ and $F_L$ measurement expected from this ongoing analysis. If one assumes the same trigger settings and the same cut efficiency, the expected number of events in each $q^2$ bin can be calculated as

$$N(q^2)_{\text{exp}}^{\text{sig}} = k \cdot \frac{L_{2012}}{L_{2011}} \cdot N_{\text{data}}^{\text{sig}}(q^2),$$

(6.1)

where $\frac{L_{2012}}{L_{2011}} = \frac{20.3 \text{ fb}^{-1}}{4.9 \text{ fb}^{-1}}$ is the ratio of the integrated luminosities, the factor $k \approx 1.2/1.1 = 1.091$ accounts for the increased $b \bar{b}$-production cross section (the numbers are taken from Figure 2.11) and $N_{\text{data}}^{\text{sig}}(q^2)$ is the measured number of signal events listed in Table 6.1. The ratio of signal and background events is assumed to stay the same as in the data of the year 2011. Toy datasets were generated using the estimated numbers of signal and background events, and the statistical uncertainty coming from the fit to these datasets was taken as an estimate for the uncertainty expected from the 2012 data analysis. The main difference introduced compared to the 2011 analysis is the usage of simultaneous instead of the sequential fit (i.e. the parameters of the mass distribution were allowed to vary).
The result of this study is summarized in Table 6.2, where \( \sigma(\mathcal{A}_{FB}) \) and \( \sigma(\mathcal{F}_L) \) denote the expected statistical uncertainty on the values of \( \mathcal{A}_{FB} \) and \( \mathcal{F}_L \), respectively. Statistical uncertainties obtained in the year 2011 are included for comparison. The expected improvement of the uncertainty is in the order of 50%, depending on the \( q^2 \)-bin. In addition, the measurement in the lowest \( q^2 \)-bin is expected from the analysis of 2012 data. The possible inclusion of the S-wave in the fit and a hopefully better understanding of the background composition is not taken into account in this study, it only estimates the improved precision due to the increased amount of the data.

Table 6.2: Expected precision of \( \mathcal{A}_{FB} \) and \( \mathcal{F}_L \) for the analysis of the data collected in the year 2012 compared to the precision achieved in 2011.

<table>
<thead>
<tr>
<th>( q^2 ) bin</th>
<th>0.04-2.00</th>
<th>2.00-4.30</th>
<th>4.30-8.68</th>
<th>10.09-12.86</th>
<th>14.18-16.00</th>
<th>16.00-19.00</th>
<th>1.00-6.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{\text{sig}} )</td>
<td>117</td>
<td>86</td>
<td>398</td>
<td>624</td>
<td>145</td>
<td>673</td>
<td>190</td>
</tr>
<tr>
<td>( N_{\text{bckg}} )</td>
<td>130</td>
<td>374</td>
<td>1730</td>
<td>567</td>
<td>906</td>
<td>701</td>
<td>826</td>
</tr>
<tr>
<td>( \sigma(\mathcal{A}<em>{FB})</em>{2012} )</td>
<td>0.09</td>
<td>0.11</td>
<td>0.05</td>
<td>0.04</td>
<td>0.09</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>( \sigma(\mathcal{F}<em>L)</em>{2012} )</td>
<td>0.07</td>
<td>0.11</td>
<td>0.06</td>
<td>0.03</td>
<td>0.08</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>( \sigma(\mathcal{A}<em>{FB})</em>{2011} )</td>
<td>-</td>
<td>0.28</td>
<td>0.13</td>
<td>0.09</td>
<td>0.19</td>
<td>0.10</td>
<td>0.20</td>
</tr>
<tr>
<td>( \sigma(\mathcal{F}<em>L)</em>{2011} )</td>
<td>-</td>
<td>0.18</td>
<td>0.11</td>
<td>0.09</td>
<td>0.16</td>
<td>0.08</td>
<td>0.15</td>
</tr>
</tbody>
</table>
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Appendices
Appendix A

Transversity Amplitudes

Under the approximation \( \frac{m_e^2}{q^2} \approx 0 \), in the SM the angular coefficients \( J_i \) can be represented in terms of transversity amplitudes \( A_i \) as [48]

\[
\begin{align*}
\frac{4}{3} J_i^1 & = \frac{3}{4} \left[ |A_{L\perp}^1|^2 + |A_{L\|}^1|^2 + (L \rightarrow R) \right], \\
\frac{4}{3} J_i^2 & = |A_{R}^2|^2 + |A_{R\|}^2|^2, \\
\frac{4}{3} J_i^3 & = \frac{1}{4} \left[ |A_{L\perp}^1|^2 + |A_{L\|}^1|^2 + (L \rightarrow R) \right], \\
\frac{4}{3} J_i^4 & = -\left[ |A_{L}^0|^2 + |A_{R\|}^0|^2 \right], \\
\frac{4}{3} J_i^5 & = \frac{1}{2} \left[ |A_{L\perp}^1|^2 - |A_{L\|}^1|^2 + (L \rightarrow R) \right], \\
\frac{4}{3} J_i^6 & = \frac{1}{\sqrt{2}} \text{Re} \left[ A_{L}^0 A_{L\|}^1 \right] + (L \rightarrow R), \\
\frac{4}{3} J_i^7 & = \sqrt{2} \text{Re} \left[ A_{L}^0 A_{L\perp}^1 - (L \rightarrow R) \right], \\
\frac{4}{3} J_i^8 & = 2 \text{Re} \left[ A_{L\perp}^1 A_{L\|}^1 - (L \rightarrow R) \right], \\
\frac{4}{3} J_i^9 & = 0, \\
\frac{4}{3} J_i^10 & = \sqrt{2} \text{Im} \left[ A_{L}^0 A_{L\perp}^1 - (L \rightarrow R) \right], \\
\frac{4}{3} J_i^11 & = \frac{1}{\sqrt{2}} \text{Im} \left[ A_{L}^0 A_{L\perp}^1 + (L \rightarrow R) \right], \\
\frac{4}{3} J_i^12 & = \text{Im} \left[ A_{L\perp}^1 A_{L\|}^1 + (L \rightarrow R) \right].
\end{align*}
\]

Non-negligible effects from finite lepton masses arise only in BSM scenarios.

Under the presence of additional operators, the expressions for angular coefficients (Eq. A.2) become
more complicated [48]:

\[
\frac{4}{3} J^i = \frac{2 + \beta_1^2}{4} \left[ |A^\perp_{L}|^2 + |A_{||}|^2 + (L \to R) \right] + \frac{4m^2}{q^2} \text{Re}(A^\perp_{L} A^\perp_{L}^* + A_{||} A_{||}^*) \\
+ 4\beta_1^2 (|A^\perp_{L}|^2 + |A_{||}|^2) + 4(4 - 3\beta_1^2) (|A^\perp_{L}|^2 + |A_{||}|^2) \\
+ 8\sqrt{2} \frac{m_1}{\sqrt{q^2}} \text{Re} \left[ (A^\perp_{L} + A_{||}) A^\perp_{L} - (A^\perp_{L} + A_{||}) A^\perp_{L} \right], \\
\frac{4}{3} J^5 = |A^\perp_{L}|^2 + |A_{||}|^2 + \frac{4m^2}{q^2} \left[ |A_{L}|^2 + 2\text{Re}(A^\perp_{L} A^\perp_{L}^*) \right] + \beta_1^2 |A_{||}|^2 \\
+ 8(2 - \beta_1^2) |A_{||}|^2 + 8\beta_1^2 |A_{||}|^2 + 16 \frac{m_1}{\sqrt{q^2}} \text{Re}((A^\perp_{L} + A_{||}) A^\perp_{L}), \\
\frac{4}{3} J^6 = \frac{\beta_1^2}{2} \left[ |A^\perp_{L}|^2 - |A_{||}|^2 + (L \to L) - 16(|A_{L}|^2 + |A_{||}|^2 + |A_{||}|^2) \right], \\
\frac{4}{3} J^7 = \frac{\beta_1^2}{2} \left[ |A^\perp_{L}|^2 - |A_{||}|^2 + (L \to R) + 16(|A_{L}|^2 - |A_{||}|^2 + |A_{||}|^2 - |A_{||}|^2) \right], \\
\frac{4}{3} J^8 = \frac{\beta_1^2}{2} \text{Re} \left[ A^\perp_{L} (A^\perp_{L})^* + (L \to R) - 8\sqrt{2} \left( A_0 A_{||} + A_{||} A_0^* \right) \right], \\
\frac{4}{3} J^9 = \sqrt{2} \beta_1 \text{Re} \left[ A^\perp_{L} A^\perp_{L}^* - (L \to R) - 2\sqrt{2} A_{L} A_{L}^* - \frac{m_1}{\sqrt{q^2}} \left( |A^\perp_{L}| + A_{||} \right) A^\perp_{L} \\
+ 4\sqrt{2} A_0 A_{||} A^\perp_{L} + 4\sqrt{2} A_0 A_{L} A_{L}^* - 4|A^\perp_{L}| - 4A^\perp_{L} A_{L}^* A_{L} \right] \\
, \\
\frac{4}{3} J^5 = 2\beta_1 \text{Re} \left[ A^\perp_{L} (A^\perp_{L})^* - (L \to R) + 4\sqrt{2} \frac{m_1}{\sqrt{q^2}} \left( |A^\perp_{L}| + A^\perp_{L} |A_{||}|^2 - |A^\perp_{L}| - A_{||} A_{||}^* \right) \right], \\
\frac{4}{3} J^6 = 4\beta_1 \text{Re} \left[ 2A_0 A^\perp_{L} + \frac{m_1}{\sqrt{q^2}} \left( |A^\perp_{L}| + A_{||} \right) A^\perp_{L} + 4A_{||} A^\perp_{L} \right], \\
\frac{4}{3} J^7 = \sqrt{2} \beta_1 \text{Im} \left[ A^\perp_{L} (A^\perp_{L})^* - (L \to R) + 2\sqrt{2} A_{L} A_{L}^* + \frac{m_1}{\sqrt{q^2}} \left( |A^\perp_{L}| + A_{||} \right) A^\perp_{L} \\
+ 4\sqrt{2} A_0 A_{L} A_{L}^* + 4\sqrt{2} A_0 A_{L} A_{L}^* - 4|A^\perp_{L}| - 4A^\perp_{L} A_{L}^* A_{L} \right], \\
\frac{4}{3} J^8 = \frac{\beta_1^2}{\sqrt{2}} \text{Im} \left[ A^\perp_{L} (A^\perp_{L})^* + (L \to R) \right], \\
\frac{4}{3} J^9 = \beta_1 \text{Im} \left[ A^\perp_{L} (A^\perp_{L})^* + (L \to R) \right], \\
\text{with} \\
\beta_1 = \sqrt{1 - \frac{m^2}{q^2}}. \\
\text{(A.3)}
\]

Here $A_0$, $A_\perp$, and $A_{||}$ correspond to the SM contributions, while all other $A_i$ originate from BSM operators, with the exception of $A_{L}$, that contributes in power $\frac{m_1}{\sqrt{q^2}}$ in the SM.
Appendix B

Trigger Description

Triggers used in the analysis:

- **EF\_mu4mu6\_DiMu** - topological trigger looking for two muons with $p_T(\mu_1) > 4$ GeV and $p_T(\mu_2) > 6$ GeV and $1.5$ GeV < $m(\mu^+\mu^-) < 14.0$ GeV

- **EF\_mu4Tmu6\_DiMu** - same as **EF\_mu4mu6\_DiMu** but the cuts on $p_T$ of the muons at L1 are tighter

- **EF\_2mu4\_DiMu** - topological trigger looking for two muons with $p_T(\mu_1) > 4$ GeV and $p_T(\mu_2) > 4$ GeV and $1.5$ GeV < $m(\mu^+\mu^-) < 14.0$ GeV

- **EF\_2mu4T\_DiMu** - same as **EF\_mu2mu4\_DiMu** but the cuts on $p_T$ of the muons at L1 are tighter

- **EF\_2mu6\_DiMu** - topological trigger looking for two muons with $p_T(\mu_1) > 6$ GeV and $p_T(\mu_2) > 6$ GeV and $2.5$ GeV < $m(\mu^+\mu^-) < 4.3$ GeV

- **EF\_2mu4\_Bmumux** - topological trigger looking for two muons with $p_T(\mu_1) > 4$ GeV and $p_T(\mu_2) > 4$ GeV and $2.5$ GeV < $m(\mu^+\mu^-) < 4.3$ GeV

- **EF\_2mu4T\_Bmumux** - same as **EF\_2mu4\_Bmumux** but the cuts on $p_T$ of the muons on Level 1 are tighter

- **EF\_mu18** - simple trigger looking for a muon with $p_T(\mu) > 18$ GeV

- **EF\_mu18\_MG\_medium** - same as **EF\_mu18** but using alternative muon reconstruction and softer cut on the $p_T(\mu)$

- **EF\_mu18\_L1J10** same as **EF\_mu18** but requiring additional jet at L1
- **EF\_mu6\_Jpsimumu** - TrigDiMuon trigger requiring one of the muons to have $p_T(\mu_1) > 6$ GeV and $2.5 < m(\mu^+\mu^-) < 4.3$ GeV

- **EF\_mu6\_Jpsimumu\_tight** - same as **EF\_mu6\_Jpsimumu** but with tighter cuts on the vertex quality of J/ψ reconstruction

- **EF\_mu10\_Jpsimumu** - TrigDiMuon trigger requiring one of the muons to have $p_T(\mu_1) > 10$ GeV and $2.5 < m(\mu^+\mu^-) < 4.3$ GeV

- **EF\_mu10\_mu10\_EFFS\_medium** - di-muon trigger looking for two muons with leading muon $p_T(\mu) > 15$ GeV and the second muon at **EF** in full ID region with $p_T(\mu) > 10$ GeV
Appendix C

Acceptance Maps
Figure C.1: Acceptance corrections for the $q^2$ region between $0.045\,\text{GeV}^2$ and $2.00\,\text{GeV}^2$

Figure C.2: Acceptance corrections for the $q^2$ region between $2.00\,\text{GeV}^2$ and $4.30\,\text{GeV}^2$

Figure C.3: Acceptance corrections for the $q^2$ region between $4.30\,\text{GeV}^2$ and $8.68\,\text{GeV}^2$

Figure C.4: Acceptance corrections for the $q^2$ region between $10.09\,\text{GeV}^2$ and $12.86\,\text{GeV}^2$
Figure C.5: Acceptance corrections for the $q^2$ region between 14.18 GeV$^2$ and 16.00 GeV$^2$

Figure C.6: Acceptance corrections for the $q^2$ region between 16.00 GeV$^2$ and 19.00 GeV$^2$

Figure C.7: Acceptance corrections for the $q^2$ region between 1.00 GeV$^2$ and 6.00 GeV$^2$
Appendix D

Fit Results
Figure D.1: Fit of $B^0_d$ candidates mass (top), $\cos\theta_K$ (middle) and $\cos\theta_l$ (bottom) in $q^2$ range between 2.00 GeV$^2$ and 4.30 GeV$^2$ [6].
Figure D.2: Fit of $B^0_d$ candidates mass (top), $\cos\theta_K$ (middle) and $\cos\theta_L$ (bottom) in $q^2$ range between 4.30 GeV$^2$ and 8.68 GeV$^2$ [6].
Figure D.3: Fit of $B^0_d$ candidates mass (top), $\cos\theta_K$ (middle) and $\cos\theta_L$ (bottom) in $q^2$ range between 10.09 GeV$^2$ and 12.86 GeV$^2$ [6].
Figure D.4: Fit of $B^0_d$ candidates mass (top), $\cos\theta_K$ (middle) and $\cos\theta_\ell$ (bottom) in $q^2$ range between 14.18 GeV$^2$ and 16.00 GeV$^2$ [6].
Figure D.5: Fit of $B_d^0$ candidates mass (top), $\cos\theta_K$ (middle) and $\cos\theta_l$ (bottom) in $q^2$ range between 16.00 GeV$^2$ and 19.00 GeV$^2$ [6].
Figure D.6: Fit of $B^0_d$ candidates mass (top), $\cos\theta_K$ (middle) and $\cos\theta_l$ (bottom) in $q^2$ range between 1.00 GeV$^2$ and 6.00 GeV$^2$ [6].
List of Abbreviations

ALICE  A Large Ion Collider
AOD   Analysis Object Data
ATLAS A Toroidal LHC Apparatus
BSM   Beyond the Standard Model
CASTOR CERN Advanced STORage Manager
CDF   Collider Detector at Fermilab
CERN  European Organization for Nuclear Research
CKM   Cabibbo-Kobayashi-Maskawa
CMS   Compact Muon Solenoid
CP    Charge Parity Symmetry
CSC   Cathode Strip Chambers
DAQ   Data Acquisition System
DIS   Deep Inelastic Scattering
DPD   Derived Physics Data
EF    Event Filter
EFT   Effective Field Theory
EM    Electromagnetic
ESD   Event Summary Data
**FATRAS**  Fast ATLAS Track Simulation

**FCNC**  Flavor Changing Neutral Currents

**FCal**  Forward Calorimeter

**FONLL**  Fixed Order plus Next-to-Leading Logarithms

**GEANT4**  GEometry ANd Tracking

**GIM**  Glashow-Iliopoulos-Maiani

**HEC**  Hadronic Endcap Calorimeter

**HQET**  Heavy Quark Effective Theory

**HLT**  High Level Trigger

**ID**  Inner Detector

**ISF**  Integrated Simulation Framework

**L1**  Level-1

**L2**  Level-2

**LAr**  Liquid Argon

**LHC**  Large Hadron Collider

**LHCb**  Large Hadron Collider beauty

**MC**  Monte-Carlo

**MDT**  Monitored Drift Tube

**MS**  Muon Spectrometer

**MSSM**  Minimal Supersymmetric Standard Model

**OPE**  Operator Product Expansion

**PDF**  Parton Density Function

**PDG**  Particle Data Group

**PS**  Prescale Factor
PSB  Proton Synchrotron Booster
QCD  Quantum Chromodynamics
QED  Quantum Electrodynamics
QFT  Quantum Field Theory
RAW  Raw data
RDO  RAW Data Object
RICH  Ring Imaging Cherenkov
RoI  Region of Interest
RPC  Resistive Plate Chambers
SCT  Semiconductor Tracker
SM  Standard Model
SPS  Super Proton Synchrotron
SUSY  Super Symmetry
TGC  Thin Gap Chambers
TMVA  Toolkit for Multivariate Analysis
TRT  Transition Radiation Tracker
WLCG  Worldwide LHC Computing Grid
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